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THEORY OF PROPAGATION OF VERY LONG WAVES

By

P. Ye. Krasnushkin and N. A. Yablochkin

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ABSTRACT: ~~This is claimed to be the first conscious~~ An attempt is made to match experimental and theoretical data on long-distance propagation of superlong (wave-lengths of several times ten kilometers) waves around the earth. Since there is no probability distribution function for the experimental data, the matching is carried out approximately by the method of mixed initial data, where all the data on the field and on the medium are divided into two groups - reliable and unreliable. Several models are proposed for the propagation along the earth's surface and in the ionosphere with an attempt to include all the geophysical factors which influence the far field of superlong radio waves. Only the waveguide channel adjacent to the earth is considered. The method of normal waves which can be used to solve waveguide propagation problems for sound waves in the ocean, infrasound waves in the atmosphere, seismic waves in the earth, etc. is also developed. The first edition was published in 1956. English translation: 103 pages.

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH

DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

PREFACE TO SECOND EDITION

1. More than seven years have passed since first publication of this book, and it went out of print in 1956. In order to decide question about expediency of its republication one should consider material contained in light of new works on propagation of very long radio waves and related geophysical problems and should also examine changes which leading physical concepts and mathematical methods that are basis of this book have undergone during the last few years.

2. From contemporary point of view [1] problem of theoretician studying propagation of radio waves under terrestrial conditions consists in coordination of experimental data on field of waves and geophysical data essentially affecting radio waves with help of functional relationships between these data. In order that theory be useful for practical workers, functional relationships between field and medium have to be derived from equations of field and material equations of medium and not directly from experimental data. These relationships establish one-to-one correspondence between functions characterizing medium and functions describing electromagnetic fields. In ideal case, when experimental data are in the form of distribution function of probability, determining degree of authenticity of experimental data, coordination consists in imposition on arguments of this function of theoretical functional relationship, which transforms unconditional probability into conditional. Thus is obtained gain of information according to Shannon, since initial regions of uncertainty of experimental data narrow. Similar state of affairs exists in statistical theory of information transmission when a priori knowledge of information is definitized according to data on signal carrying the information [2]. Let us note that in process of coordination of data there appears gain of information

both for field and for medium. Its magnitude depends on character of functional relationship and degree of authenticity of experimental data. As it is known, information may be obtained only from experiment. In this case no additional experiments are carried out and gain of information is due to experimental data included in theoretical functional relationships. Progress of theory consists in that owing to discovery of new facts, more precise definition of old, or desire to include known facts in realm of coordination, there occurs from time to time recoordination of data on field and medium on the basis of more exact or broader functional relationships.

This book, as far as I know, constitutes first conscious attempt to apply concepts of coordination to problem of long-range radio communications on kilometer waves (very long) around Earth. Owing to absence of distribution functions of probability for experimental data, coordination was produced by approximation method of mixed initial data. It consists in separation of all data on field and medium into two groups: reliable and unauthentic data. First are introduced into functional relationships as initial data, while second are determined from these relationships as unknown quantities. Method of mixed initial data gives certain loss of information as compared to ideal method of coordination, since in it information contained in unlikely data is disregarded and reliability of reliable data introduced as initial is reevaluated. However, this loss is not so big as obtained during solution of primal problems of propagation of radio waves for which data on field are generally disregarded and all data on medium are considered absolutely reliable. For problem of propagation of very long radio waves around Earth there is obtained in this case negative value for gain of information according to Shannon [1].

3. Data obtained as a result of coordination will have meaning of new information on field and medium only under the condition that functional relationships are adequate to true ones. There was possibility of obtaining uncontradictory coordination of limited number of experimental data says nothing for adequacy of functional relationships. Coordination can also be obtained with incorrect functional relationship, if it contains sufficient number of arbitrary constants. As example of this we have numerous phenomenological formulas for calculation of distant field of radio waves (Osteen, Ekspenshid, and others) appearing till now in many textbooks and monographs on propagation of radio waves. Another method, creating illusion of adequacy of functional relationships, consists in that some of parameters

of medium or field are declared effective, and it is permitted to take different values, depending upon what group of facts is subjected to coordination. Such method was used during the study of lower ionosphere [9-11] (points 4 and 5 this preface), which in recent past was still in the full sense a blank spot in geophysics, and it was possible to ascribe any properties to it. Necessity to change parameters of ionosphere during transition fron one particular coordination of data to another is criterion of inadequacy of model of medium. This criterion was used in present work during construction of models A and B (Table 1). However, basic method of selection of correct model of medium, requiring enlistment of minimum quantity of experimental data, consisted in the following. Having assigned a group of facts to be subjected to coordination and having selected certain model of medium, we estimated what parameters of model essentially influence these facts through functional relationships. Gradually complicating model of medium and constructing functional relationships between field and medium for it, we developed simplest models of medium A and B (Table 1), considering all basic geophysical factors essentially affecting properties of distant field of very long radio waves. As a result of such analysis we determined necessity for calculation of sphericity of earth and ionosphere, conductivity of earth, magnetic anisotropy of ionosphere, and also changeability of medium along route of radio wave. Lower part of ionosphere was replaced by step model, characterizing concentration of electrons and effective frequency of collisions of electrons with neutral molecules. Possibility of disregarding motion of ions in lower ionosphere, in spite of their large quantity in this region, follows from works of Bates, Messi, and Nicolet. Effective frequency of collisions was determined from Nicolet's theory, which, as recent investigations showed [3], gives somewhat exaggerated values in connection with fact that transport cross section for molecules of nitrogen is proportional to velocity of electrons and not inversely proportional to it as was assumed by Nicolet. Doubt can also be caused by replacement of real smooth distributions of electron concentration and frequency of collisions with height by one- or two-stage curves. However, more general theory of propagation of long, very long, and extremely long radio waves around Earth [1, 4] showed that such idealization is permissible for waves with frequency $f < 30$ kilocycles and distances $D > 500$-1000 km. In these works it was shown that for expansion of functional relationship to region of data embracing frequency up to 100 kc and any distances, introduction of several steps

is inexpedient. More effective results are given by smooth model of electron concentration and frequency of collisions. In these works was also clarified unreasonableness of use as initial data of measurements of near field only. This is connected with fact that on long-range routes there can be considerable local heterogeneities, caused by geoactivity of Sun and radiation belts of Earth. Thus appraisal of possibility of predictions of properties of distant field according to data on near field only is too optimistic.

4. Consideration of problem in light of information theory has obscured the semantic side of question. At the same time, as a result of coordination with help of adequate functional relationships between field and medium, the mechanism of long-range propagation of very long radio waves around Earth is clarified. As follows from Chapter 2, distant field is in the form of spectrum of normal waves [5, 8] propagating in unique waveguide shaped like surface of earth and lower layers of ionosphere. These waves are similar to TH and TE waves in hollow metallic pipes, known since times of Rayleigh [6]. However, Watson, having obtaining in 1919 [7] a field of long waves in the form of spectrum of normal waves, did not discover relationship of propagation in hollow pipes to waveguide mechanism. Method of normal waves [5, 8] utilized in this book establishes this relationship automatically. Therefore we use waveguide terminology throughout. Thanks to existence of critical frequencies for all waves but TH_0, determined by ratio of width of waveguide to wavelength, over large distance of field of radio wave, there are not more than two-three normal waves. It is necessary to note that under real conditions, besides surface waveguide, there are underground waveguide, through which travel normal waves, creating above earth so-called lateral waves (terminology of Ott and L. M. Breknovskikh), and ionospheric waveguides, among which is separated waveguide channel of whistling atmospherics [sic]. General diagram of spectra of normal waves is analyzed in works [1, 4]. In this book only surface waveguide channel is considered.

We direct attention of reader to necessity for calculation of magnetic field of Earth, sphericity of waveguide walls, and finite conductivity lower wall of Earth. Magnetic anisotropy of ionosphere, appearing in night, cancels Brewster effect. Without it distant field of very long waves would be weaker at night. Disregard for sphericity of Earth and ionosphere at very long lengths, and all the more so at long wavelengths, is absolutely impermissible. This is connected with existence of effect of adhesion of waves of any nature to concave surfaces: in this

case, to lower ionosphere. Effect of adhesion was discovered with acoustic waves by Rayleigh; with radio waves it was investigated in work [5, 8]. Normal waves in which effect of adhesion appears have angular wave numbers lying in interval between ka and kc, c = a + h; a = 6370 km. This means that they possess phase velocities less than velocity of light, as can be seen from Fig. 17. In waveguide with flat walls phase velocities of normal waves are always greater than velocity of light, unless we ascribe supernatural properties to earth or ionosphere. Adhesion of normal waves to ionosphere considerably weakens coefficients of excitation of normal waves n_j (see Fig. 19) as compared to flat case. Comparisons of flat and spherical waveguide are given in Fig. 3 of work [1]. With increase of frequency, besides effect of adhesion, there occurs weakening of reflectance of ionosphere, and, as a result of these two effects, mechanism of propagation of waves in continuous manner passes from waveguide to diffraction. This transition is accompanied by replacement of normal waves, dominating at large distance from radiator. Thus, for instance, for extremely long waves ($f < 3000$ cps) in distant field TH_0 wave dominates, also called TEM or cabin wave. In range of frequencies from 10 to 20 kilocycles, considered in this book, there dominates, as a rule, the TH_1 wave, while in certain cases the TH_2 is dominant. At frequencies of 50-60 kilocycles by day TH_2 and TH_3 waves dominate, and at night the TH_4 and TH_5 are dominant [1, 4]. As a result of replacement of leading waves average phase velocity along Earth at large distances, determined by leading wave, is always greater than velocity of light for $f > 1000$-2000 cps. Diurnal variations of distant field of very long and long waves are determined by interference and replacement of above-indicated normal waves. Finally, in present work is established influence of finite conductivity of earth on attenuation factors of leading normal waves (see Fig. 15). For "soil" of average conductivity they are increased by approximately 0.5 neper per radian of distance.

5. However, attempts continue to insert certain experimental data on very long waves into the Procrustean bed of functional relationships derived from simplified models of medium in which are disregarded sphericity of earth and ionosphere, conductivity of earth, magnetic anisotropy of ionosphere, and changeability of medium along route of wave. Impossibility of uncontradictory coordination with help of these functional relationships follows in certain cases from works themselves. Thus, for instance, in works [9, 10, 11] the author had to change height of lower layer of ionosphere from 70 to 50 km and its conductivity in

range 10^4 - 10^2 CGSF, depending upon selected frequency and distance of route, which, as noted above, is first criterion of inadequacy of model and was used by us for its correction. Thus formulas obtained by them differ little from phenomenological formulas, thanks to contribution of Osteen, Ekspenshid, and others. The route to such simplification was taken also by certain foreign authors [20 and 21]. Only recently have works considering some of esstential factors separately began to appear [12, 13]. From Chapters IV and V of this book it follows that these factors interact with each other and that their calculation separately cannot bring benefit to practice. Thus, for instance, calculation of conductivity of earth in flat model yields exaggerated attenuation factors [20], since effect of adhesion weakens influence of earth.

6. From the point of view of contemporary state of problem of propagation of very long radio waves around Earth, in this book the following results are the most important: 1) explanation of diurnal variations of distant field of very long waves; 2) prediction of diurnal variations in phase of distant field, confirmed completely by experiments of Pierce and Heritage in the United States [14, 15], and selection of the most phase-stable range of waves useful for transmission of signals of exact time; 3) appraisal of electron concentration of lower ionosphere by day and at night. With respect to the latter, one should note that replacement of smooth distribution of profile of electron concentration by stepped type permits us to treat equivalent parameters only. In reality, as was shown in works [1, 3, 4], lower ionosphere consists of three layers, C, D, and E. Layer C plays role of absorbing pad, weakening reflectance of layer D or E. It was discovered as result of coordination of data on field and medium by method of mixed initial data. This was done in autumn of 1959 and reported at Colloquium of VTs, Academy of Sciences of USSR on 17 March 1960. I take this opportunity to express gratefulness to candidate of physical and mathematical sciences, S. P. Lomnev, who composed complicated program of calculations on BESM-1 and carried them out in short period.

7. Concerning method of normal waves, we note that its application in this book, when wave numbers of normal waves are complex and spectra are mixed, would have been impossible without spectral theory of singular nonself-conjugate operators so strongly advanced during the last few years by Soviet mathematicians [16, 17, 18]. Now by method of normal waves presented in Chapter II can be solved problem of waveguide propagation of sound waves in ocean, infrasonic waves in atmosphere, seismic waves in earth, etc. Until now these problems, like that considered in this

book, were solved by quite complicated artificial method (Watson method [7], method of mirror images), in which normal waves appeared without relationship to waveguide mechanism of wave propagation. Moreover, in certain cases errors appeared. For instance, in work [19] were introduced waves TH_j and TE_k with negative indices j and k. Besides, in the same place and then and in works [9, 10, 11] were united TH_0 and TH_1 waves. In case of application of method of normal waves such errors are practically impossible. We expended much effort in order to simulate conditions under which negative j and k appear. For this it turned out to be necessary to violate principle of radiation with respect to waves traveling on radius r. Impossibility of displacement of TH_0 and TH_1 waves follows from Fig. 10 and also is due to distinction in their forms in ideal case, when walls of waveguide have infinite conductivity.

Summarizing what has been said, one should recognize what book holds interest for specialists on wave propagation even at present. At the same time inclusion in it of new ideas expressed in this preface would demand radical alternation of a number of its chapters. Therefore it was resolved to republish it by sterotype, using photomechanical method.

25 November 1962

After this preface was written, leading specialist on propagation of very long radio waves, J. White, sent me copies of his numerous works (see references [22-27]), illuminating contemporary state of art in the United States. From these works we find that starting from 1960 he rejected flat isotropic model of medium used in works [20, 21] and began to consider sphericity and magnetic field of Earth approximately as this is done in our book. However, he applied Watson's classical method of calculation of field [7], which is not connected with contemporary spectral theory of linear operators [16, 18], and results obtained by this method pertaining to uniform routes (day), without explaining daily variation of very long wave [VLW] (СДВ) field, in no way correspond to results obtained in this book or to method of calculation of fields (method of normal waves) used. Limitedness of results obtained in works [22-27] also is connected with fact that their author did not attempt to coordinate all known data on field and medium and disregarded factual information on field of waves, which prevented him from constructing general picture of VLW propagation under terrestrial conditions. All these circumstances once again confirmed my confidence in expediency of republication of the book.

Author of preface trusts that methods offered for coordination of data, method of normal waves, and method of modulated normal waves, which were used by him for formulation of theory VLW under terrestrial conditions, will find application in other stratified and approximately stratified media. He also takes opportunity to express gratitude to N. A. Yablochkin, who developed a number of numerical methods for calculation of parameters of normal waves and directed calculation group which obtained numerical results given below.

5 October 1963
P. Ye. Krasnushkii

Literature

1. P. Ye. Krasnushkii. DAN Vol. 139, 1961, No. 1.

2. K. Shannon. In collection, "Theory of transmission of electrical signals in the presence of interferences," Moscow, Izd-vo in. lit., 1953.

3. P. Ye. Krasnushkin and N. L. Kolesnikov. DAN Vol. 146, 1962, No. 8.

4. P. Ye. Krasnushkin. "Fiz, ents. slovar," Vol. I, 1960. "Atmosferiki;" DAN Vol. 138, 1961, No. 4 and 5; DAN, Vol. 140, 1961, No. 4; Dan, Vol. 144, 1962, No. 2; Nuovo Cimento, Sup., Vol. 26, SX, 1962.

5. P. Ye. Krasnushkin. Method of normal waves, doctoral dissertation, Moscow State University, November 1945; "Fiz. ents. slovar," 1963, Vol. III "Normal waves."

6. B. A. Vvedenskiy and A. G. Arenberg. Radio waveguides, Part 1, Moscow-Leningrad, Gostekhizdat, 1946.

7. G. N. Vatson. Proc. Roy. Soc. A, Vol. 95, 1919, page 546.

8. P. Ye. Krasnushkin. Method of normal waves in application to problem of long-range radio communications, Moscow State University, 1947.

9. Ya. L. Al'pert. On propagation of low-frequency electromagnetic waves of above earth's surface, Moscow, Izd-vo Academy of Sciences of USSR, 1955.

10. Ya. L. Al'pert. Y. F. N. Vol. 60, 1956, Issue 3, page 370; Radio engineering and electronics, 1956, No. 3, page 281.

11. Ya. L. Al'pert. Propagation of radio waves and the monosphere, Chapter 7, Moscow, Izd-vo Academy os Sciences of USSR, 1960.

12. K. G. Budden. Proc. Roy. Soc. A, No. 323, Vol. 265, 1962, page 538.

13. K. G. Budden and Kh. G. Martin. Ibid, page 554.

14. J. A. Pierce. PIRE, Vol. 43, 1955, page 584; Vol. 45, 1957, page 794.

15. D. P. Heritage and others. PIRE, Vol. 47, 1959, page 829.

16. M. V. Keldysh. DAN, Vol. 77, 1951, page 1.

17. M. A. Naymark. Transactions of Moscow Mathematical Society, Vol. 3, 1954, page 181.

18. V. A. Marchenko. Mathematical collection, 52, 1960, Issue 2, page 739.

19. K. G. Budden. Phil. Mag., Vol. 42, 1951, page 1; Vol. 43, 1952, page 179; Vol. 44, 1953, page 504.

20. J. White. PIRE, Vol. 45, 1957, pages 761 and 768.

21. A. G. Ginn, V. L. Teylor, and J. White. Journal of Geophysical Research, 1960, Vol. 65, No. 3, 907.

22. J. White and K. Spice. Journal of Geophysical Research, 1960, Vol. 65, No. 8, August, 2325-2331.

23. J. White. Journal of Research of NBS, 1961, Vol. 67D, No. 1 January-February, 37-46.

24. J. White and K. Spice. Journal of Research of NBS, 1963, Vol. 67D, No. 2, March-April, 183-187.

25. J. White. Canadian Journal of Physics, 1962, Vol. 41, 299-315.

26. J. White. "Electromagnetic waves in Stratified medium," Oxford, Program Press, 1962, 264-288.

27. J. White. PIRE, 1962, July, Vol. 50, 1624-1647.

INTRODUCTION

CONTEMPORARY STATE OF THEORY OF VERY LONG WAVE VLW PROPAGATION

Development of theory [VLW] (СДВ) propagation may be divided into two periods. Basis of first period, which began from first stages of introduction of radio into practice of long-distance communication, is diffraction concept, advanced in 1904 by Rayleigh [13].

In development of theory of propagation of long and very long waves on the basis of diffraction model, in which existence of ionosphere was disregarded and earth was considered as uniform conducting sphere, took part Poincaré [14], MacDonald [15], Nicholson [16], Sommerfeld [17], Love [18], Marx [19], Rybchinskiy [20], and others.

Concerning these investigations, in work [21], representing combined result of investigations of the first period, it is stated that, "many years of work of the most reknown mathematicians was insufficient to advance this initially single appearing problem beyond the state about which Nicholson said that of all mathematical problems it alone has caused such great divergence of opinions." Not until 1918 did Watson [22] succeed, on the basis of works [15] and [16], in making final conclusion that by the phenomenon of diffraction of long waves around globe alone it is impossible to explain fact of long-range propagation of these waves.

Second period, founded on waveguide concept, began in 1919 with work of Watson [23]. In this work as model of medium is considered uniform isotropic semiconducting globe, consisting in concentric uniform semiconducting isotropic ionosphere. Radio waves are excited by vertical Hertz doublet and propagate in spherical layer of atmosphere, located between earth and ionosphere, just as microwaves propagate in contemporary waveguides.

In development of theory of propagation of long and very long waves on the basis of spherical waveguide model with isotropic conducting walls participated, besides Watson [23], Riedback [24] and Bremmer [5], who considered sphericity of earth, and also Kenrick [25], Weirich [26], Brekhovskikh and Ryazin [27], and Budden [28] who considered it possible to simplify Watson's model by substituting radii of earth and ionosphere and that conductivity of earth is infinite.

Stumbling block of all above-indicated works of second period was transcendental equation in cylindrical functions for determination of wave numbers of elementary waves, called normal waves by us, of which is composed long-range field. During its solution were allowed different approximations of cylindrical functions, which led to considerable errors. Another problem in shown works was absence of reliable data about properties of lower layers of ionosphere, which have to be used in Watson model or its plane analog so that theory yields simple answer.

Certain authors managed to compensate defects of idealization and error in calculations through proper selection of parameters of model of ionosphere. As a result of such "preparation" was created impression that their theories were adequate to reality.

Example are works founded on spherical model of medium, in which is obtained good agreement with Osteen-Kogan empirical formula. Here the cited authors were forced to ascribe lower boundary of ionosphere of fantastically high conductivity

$$-\sigma = 1{,}44.10^{-16}\,(CGS\mu) = 1{,}3.10^{6}\,(CGS\varepsilon),$$
$$-\sigma = 4.10^{-16}\,(CGS\mu) = 3{,}6.10^{6}\,(CGS\varepsilon),$$

which is at least two orders of magnitude greater than real equivalent conductivity of very lowest layer of ionosphere, responsible for long-range VLW field in daytime. In works using plane waveguide model it was possible also to obtain coincidence with separate experimental measurements of amplitude of field by means of proper "preparation" of parameters of ionosphere.

In recent years new works have appeared in which is used plane waveguide model of medium, taking into account magnetic field of earth within the bounds quasi-longitudinal theory. Here was obtained coincidence with experimental curve of decay of long-range field in spite of errors in calculation and inadmissibility of quasi-longitudinal approximation. In these works, as in those mentioned above, two or three unknown and arbitrary parameters of ionosphere allowed authors to explain the same isolated experimental fact through different combination of numerical values of these parameters and different types of models. Such state of affairs did not

favor development of theory of VLW, and quite understandably there still exists divergence of opinions on principal characteristics of waveguide mechanism of propagation of VLW. For instance, there remain controversial the roles of sphericity of earth and ionosphere, of magnetic anisotropy of ionosphere, of erosion of lower boundary of ionosphere, of ground conductivity, and also relief of earth's surface and ionosphere. At the same time the wealth of experimental material accumulated in over 50 years of practice of long-distance communication on VLW has turned out to be outside the domain of applicability of theory and presents a quite disjointed picture. Existing theories have not been able to explain even following well-known and clear facts:

a) diurnal variations in long-range VLW field, which are striking for their regularity at every fixed point of reception and for their changeability as compared to diurnal variations at different points of reception. Thus, for instance, there remains the puzzle of 4-5-fold increase of signal strength at night on Nauen-Tokyo line [1] and Hawaian islands-Moscow line, a two-fold weakening of signal at night on Hawaian islands-Tokyo line [1], a sharp 15-20-fold weakening of night signal on Rugby-Moscow line (Chapter V, Section 3), etc;

b) seasonal changes in strength of reception of long-range VLW stations;

c) complicated dependence of field strength on distance at night and on routes of varying illuminance;

d) amplification of signals of long-range VLW stations during flashes of radiation of sun, accompanied by loss of shortwave signals.

CHAPTER I

FORMULATION OF PROBLEM AND SELECTION OF MODEL OF MEDIUM

§ 1. Mixed Problem of Theory of VLW Propagation

In spite of impossibility of using existing waveguide theories of propagation of long waves for purposes set in this work, we subjected them to detailed analysis, since it was not clear from above-cited works how and in what measure offered models of media differ from reality.

So that analysis of models is effective it is necessary to be free from two basic deficiencies of preceding theories: 1) arbitrariness of selection of data on ionosphere and 2) inaccuracy of algorithm of calculation of electromagnetic field according to data on medium.

The first of these deficiencies was removed by new formulation of problem. Usually solved is primal problem of theory of waves, where electromagnetic field is determined in accordance with assigned properties of medium. In our case properties of medium are given incompletely; therefore it is necessary to solve so-called mixed problem, for which with known and reliable, but not complete, data on medium and also from known additional data on [VLW] (СДВ) field are determined unknown data on medium and remaining VLW field.

Since we are interested in data on long-range VLW field, then as initial data of mixed problem it is expedient to take incomplete data about medium and data about short-range VLW field. Result of theory in this case will be data on long-range field of VLW and unknown data on lower layers of ionosphere.

Solution of mixed problem is carried out by following method. All reliable initial data on properties of ionosphere, atmosphere, and earth are included in model of medium, that is, determine structure of function of dielectric constant,

which depends on spatial coordinates, for instance, spherical **r**, θ, and φ and also on several parameters α, β, γ.....x, y, z.......

$$\|\bar{\varepsilon}\| = \|\bar{\varepsilon}(r,\theta,\varphi;\ \alpha,\beta,\gamma \dots\ x,y,z \dots\dots)\|. \quad (1.1)$$

Permeability is considered equal to 1 in CGS. Fact of incompleteness of data on medium is expressed in fact that for part of these parameters x, y, z ... their numerical values are unknown.

Let us assume that also at a number of points of near zone

$$(r_j, \theta_j, \varphi_j) \qquad j = 1,2,3 \dots\dots N$$

there is known from experiment some component of VLW field

$$\bar{E}_{\theta j} = |\bar{E}(r_j, \theta_j, \varphi_j)|\, e^{i\Psi(r_j,\theta_j,\varphi_j)}\, e^{-i\omega t}, \quad (1.2)$$

excited by harmonic currents

$$\bar{I} = \bar{I}(r,\theta,\varphi), \quad (1.3)$$

localized in near zone. Then by assigning I according to (1.3) and function $\|\bar{\varepsilon}\|$ according to (1.1), we solve primal problem of theory of propagation of electromagnetic waves and find component of field $\bar{E}$, as function of $\bar{I}$ and $\|\bar{\varepsilon}\|$. Let us record this solution with help of operator F

$$\bar{E}(r,\theta,\varphi) = F_{r,\theta,\varphi}[\bar{I}(r',\theta',\varphi');\ \|\bar{\varepsilon}(r',\theta',\varphi';\ \alpha,\beta,\gamma \dots\ x,y,z \dots)\|]. \quad (1.4)$$

Since current $\bar{I}(r', \theta', \varphi')$ in formula (1.4) coincides with current of formula (1.3), realized in conditions of experiment, in which were measured $|E_j|$ and Ψ_j (j = 1, 2, 3, ... N), then near field E_6, recorded in (1.2), should satisfy equation (1.4). Therefore by putting (1.3) in (1.4) we obtain 2N equations:

$$|\bar{E}_{\theta j}| = \mathrm{mod}\, F_{r_j,\theta_j,\varphi_j}[\bar{I}(r',\theta',\varphi');\ \|\bar{\varepsilon}(r',\theta',\varphi';\ \alpha,\beta,\gamma \dots x,y,z \dots)\|], \quad (1.5)$$

$$\Psi_j = \arg F_{r_j,\theta_j,\varphi_j}[\bar{I}(r',\theta',\varphi');\|\bar{\varepsilon}(r',\theta',\varphi';\ \alpha,\beta,\gamma \dots\ x,y,z \dots)\|] \quad (1.6)$$

$$j = 1,2,3 \dots\dots N$$

for determination of numerical values of parameters x, y, z, ... If number of unknown parameters x, y, z, ... is equal to M, there should be taken at least M/2 points (r_j, θ_j, φ_j), in which amplitude $|E_j|$ and phase Ψ_j of near field are known.

Solving system of equations (1.5) and (1.6), we find numerical values of x, y, z, ... and putting them in (1.4), we determine from it long-range field $E_Д$.

Solution of mixed problem offered above is depicted schematically in Fig. 1.

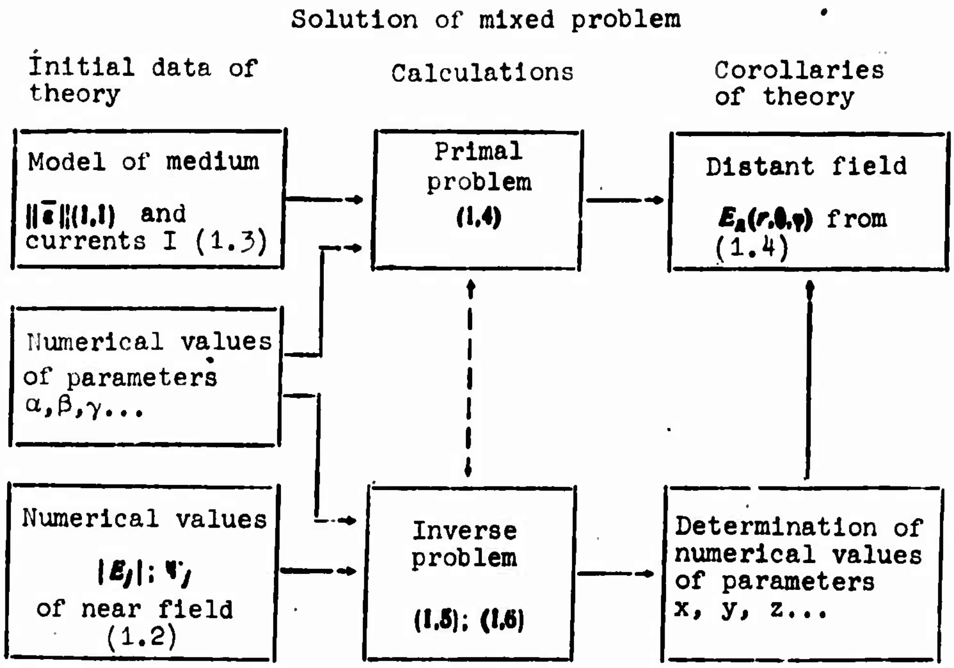


Fig. 1.

From diagram of Fig. 1 one may see that solution of mixed problem is replaced by solutions of primal and inverse problem, which are based on the same model of medium, since algorithm of inverse problem is obtained from solution of primal problem (this fact is shown by dotted pointer).

Role of primal and inverse problems in diagram of Fig. 1 is determined by ratio of volumes of initial data on medium and field E_6. If all data on medium necessary for calculation of field E are known, that is, unknown coefficients x, y, z, ... are absent, then no additional data on field E_6 need be introduced and problem is turned into primal one.

In other limiting case data on medium are completely absent; then number of unknown parameters x, y, z, ... becomes infinitely large and entire field, both near and distant, should be used as initial data of problem, which degenerates to inverse problem of theory of waves.

In intermediate case (mixed problem) number of initial data on near field, and this means actual zone of near field also from which we take these data, grows with increase of number of unknowns x, y, z, ..., characterizing degree of our ignorance of properties of medium.

Basic formula (1.4) was obtained by method of normal waves and represents sum of normal waves propagating on coordinates θ and φ (Chapter II). For calculation of wave numbers of normal waves it is possible to apply direct algebraic methods

which permit easy calculation of wave numbers with an accuracy of 4-5 digits, which is necessary for determination of properties of distant field, depending on difference of wave numbers of first types of normal waves. Until now wave numbers were determined from transcendental equation with considerably less accuracy. Thus method of normal waves permits us to remove second deficiency of preceding theories shown in beginning of this paragraph. During use of calculation diagram of Fig. 1 and with exact algorithms of calculation of near and distant fields, all divergences between corollaries of theory and experimental data on distant field can be caused only by inadequateness of model of medium.

§ 2. Analysis of Preceding Models and Selection of New Model of Medium

Method of solution of problems of theory of VLW propagation presented in § 1 places in hands of researchers effective implement for analysis of models of media. Using this method during 1950-1955, we analyzed in detail different models of media given in Table 1. In first vertical columns of this table are enumerated properties of media utilized in models. In upper line are shown types of models or authors offering and using them. To every model is allotted one vertical column in which plus signs denote considered properties and blanks or minus signs denote neglected properties enumerated in first column.

During the study of waveguide models enumerated in Table 1, we established following important internal regularities of models.

1. For calculation of near field of VLW (to 300 km) it is possible to apply flat model of medium. Number of normal waves playing essential role in near field is 1.5-2 tens; therefore representation of field of near zone in the form of sum of normal waves does not give effective method of calculation of field. More convenient in near zone is beam method, which is used in works [31, 32, 33, 34 and 35] for determination of properties of near field are explained by interference of one direct beam and 2-3 beams reflected from ionosphere and earth. Beam method can be applied also at large distances (to 1000 km), where it is necessary to consider approximately sphericity of medium.

2. For calculation of distant field of VLW (more than 2000 km) in range 10-20 kilocycles strict calculation of sphericity of earth and ionosphere is necessary, so that in distant field there dominate normal waves of first numbers, field of which is weakened near earth's surface from effect of adhesion of waves to concave surface of ionosphere [36, 37]. Small quantity of normal waves determining distant field

Table 1

			Rayleigh 1904 Watson 1918	Watson 1919	Riedbeck 1944	Bremmer 1949	Brekhovskikh and Ryazin 1946	Budden 1951	Budden 1952	Model A	Model B	Model C
Earth	Geometric character-istics	Plane					+	+	+			
		Spherical	+	+	+	+				+	+	+
		With allowance for relief	−	−	−	−	−	−	−	−	−	+
	Electrical character-istics	Ideally conducting					+	+	+			
		Semiconducting	+	+	+	+				+	+	+
		With allowance for horizontal hetero-geneities	−	−	−	−	−	−	−	−	−	+
Ionosphere	Geometric character-istics	Plane	−				+	+	+			
		Spherical	−	+	+	+				+	+	+
		With allowance for relief	−	−	−	−	−	−	−	−	−	+
	Electrical character-istics	Ideally conducting	−									
		Semiconducting	−	+	+	+	+	+	+	+	+	+
		With allowance for magnetic field of earth	−	−	−	−	−	−	+	+	+	+
		With allowance for horizontal hetero-geneities	−	−	−	−	−	−	−	−	−	+
	Character-istic of transition region	Sharp transition	−	+	+	+	+	+	+	+		
		Stepped	−								+	+
		Smooth	−		+							

makes expedient application of method of normal waves for calculation of distant field.

Application of plane models gives not only quantitative but also qualitative divergence from reality. Thus, for instance, cable wave, being basic in plane model, is practically absent in spherical for frequencies $f > 10$ kilocycles.

3. Disregard for conductivity of earth's surface is permissible only on routes passing above marine surface. On long-range routes passing above land disregard for conductivity of "soil" can give computed value of field strength 2-3 times exceeding real value.

4. Both near and distant VLW fields are essentially influenced by magnetic field of earth. Disregard for vertical component of magnetic field of earth leads to considerable weakening of reflection of VLW in region of quasi-Brewster angles as compared to observed values. Watson's model leads to 10-20-fold weakening of distant field at night as compared to day, whereas in reality there is observed 2-5-fold amplification of night field of VLW. In daytime allowance for influence of magnetic field of earth is not so essential.

Quasi-longitudinal theory of Booker [30], used in work of Budden [29], is inapplicable for calculation of influence of magnetic field of earth on distant VLW field, since with glancing angles of incidence of waves hypothesis of quasi-longitudinal character of magnetic field is not satisfied.

5. VLW's striking ionosphere at large (distant zone) and small (near zone) angles of incidence are reflected from ionosphere at different heights. Therefore in model of medium there should be considered fact of finiteness of transition region from atmosphere at different heights. Therefore in model of medium fact of finiteness of transition region from atmosphere to ionosphere should be considered.

6. For calculation of distant field on routes of varying illuminance (night — day), with varying character of earth's surface and also under conditions of changeability of ionosphere, it is necessary to allow for changes of height of ionosphere, its electrical properties, curvature of earth, and its conductivity and relief along route of radio waves connecting corresponding points.

Our problem consists in formulation of theory of VLW propagation which adequately explains on the basis of single model of medium regular phenomena of near and distant fields under conditions of propagation on routes of mixed illuminance (night — day), varying properties of earth's surface (land — sea), and also random phenomena caused by flashes on sun, by magnetic storms, and stationary turbulence of ionosphere at night.

Considering in light of these requirements imposed on theory of VLW the six above-enumerated points characterizing VLW propagation in different models, Table 1, we are convinced that in view of nonfulfillment of points 4, 5 and 6 models of media applied earlier, in particular the Watson and Budden models, cannot be used for formulation of adequate theory of VLW. In connection with this we considered new models of media, designated in Table 1 by letters A, B, and C.

Model A is generalization of Watson's waveguide model for case of calculation of vertical component of magnetic field of earth.

Model B differs from model A in that in it there is considered erosion of layer of ionosphere, which is represented in the form of series of concentric layers with radii r_k, $k = 1, 2, \ldots N$, each of which is characterized by tensor of dielectric constant $\|\bar{\varepsilon}_k\|$, remaining constant within limits of layer. Model A is particular case of model B for $N = 1$.

Model C represents further generalization in this series of models. In it is considered fact of slow changes of radii of layers r_k and their electrical properties $\|\bar{\varepsilon}_k\|$, depending upon geographic coordinates θ and φ.

From 6 above mentioned points it follows that only model C satisfies all requirements imposed on VLW theory. Model B has field of application (uniform route under night or day conditions). Model A can be applied only for approximate calculations, since it does not permit us to use method of introduction of initial data of near field as per diagram depicted in Fig. 1.

It is necessary to note that in region of near field, which we shall use in Chapter III during introduction of initial data, model C is replaced by model B. Thus as initial data on E_6 are introduced averaged quantities; therefore unknown parameters of medium r_k and $\|\bar{\varepsilon}_k\|$ obtained from theory also are averaged over distances of 300-500 km.

§ 3. Plan of Work

As basis of this work are models of media B and C described in preceding paragraph. Solution of mixed problem of theory of propagation of waves is carried out by method described in § 1. Therefore diagram depicted in Fig. 1 is essentially plan of present work.

In Chapter II per diagram of Fig. 1 are constructed models of media, and by method of normal [36, 37] waves is solved primal problem of theory of propagation of radio waves for models B and C.

In Chapter III are given results of solution of inverse problem of theory of waves, determining unknown parameters of lower layers of ionosphere x, y, z, ...

In Chapters IV-VI we return to primal problem, introducing in it known data on medium (α, β, γ, ...) and new data on ionosphere (x, y, z, ...) obtained in Chapter III.

Chapter IV contains basic characteristics of normal waves and their dependence on parameters of medium and radiator.

Chapter V is devoted to calculation of regular processes of VLW propagation over large distances.

CHAPTER II

SOLUTION OF BOUNDARY PROBLEM OF VLW PROPAGATION ON BASIS OF MODELS B AND C

§ 1. Introduction of Data on Properties of Medium Represented by Models B and C

Models B and C, briefly described in § 2, Chapter I, have so general form that they can be applied to any radio waves propagating around the globe by means of ionosphere when it is possible to disregard small-scale turbulences and horizontal component of magnetic field of earth (high and middle latitudes). If we apply models B and C in so general form for solution of mixed problem per diagram of Fig. 1, large quantity of experimental data on field E (near and distant) will be demanded, which have to be introduced as initial data of theory, so that theory yields simple answers. Here the main part of corollaries of theory will consist of information represented by models B and C, and only small part will represent distant field E. Considering that our main aim is to obtain new information about distant field E and not study of properties of lower layers of ionosphere, we should make models B and C as specific as possible by using sources of information unbound to experimental data on [VLW] (СДВ) field. Here problem will approach the primal, and volume of additional experimental information about field $E_б$, utilized as initial data of theory, will decrease. We have all data necessary to represent two first layers $(0, r_0)$ and (r_0, r_1).

Earth layer occupies interval $(0, r_0)$, where $r_0 = a$ is radius of earth's surface. Since VLW's do not penetrate earth or water by more than 1-2 km, we shall consider that whole globe possesses uniform conductivity $\sigma = \sigma_0$ and is characterized for frequency ω by complex dielectric constant.

$$\varepsilon_0 = \varepsilon_0' + \frac{i4\pi\sigma_0}{\omega}, \tag{2.1}$$

corresponding to effective value ε_0 in upper layers of earth for frequency $f = = \frac{\omega}{2\pi}$ = 10-20 kilocycles. Numerical values of $\mathbf{r}_0$ and ε_0 are well known and will be introduced later, in Chapters IV-V.

Following layer of models B and C is atmosphere ($\mathbf{r}_0$, $\mathbf{r}_1$). From data on measurement of dielectric constant in lower layers of atmosphere it follows that for VLW dielectric constant of layer of atmosphere may be considered equal to 1.

Data on layers of ionosphere ($\mathbf{r}_1$, $\mathbf{r}_2$); ($\mathbf{r}_2$, $\mathbf{r}_3$); ($\mathbf{r}_3$, $\mathbf{r}_4$)......($\mathbf{r}_{N-1}$, ∞) are scanty and little reliable. From literature known to us about lower layers of ionosphere we are unable to obtain reliable quantitative data about radii $\mathbf{r}_k$ and tensors $\|\varepsilon_k\|$ ($k = 2, 3, \ldots N$). Together with this, from these sources it is possible to represent structure of ionospheric layers, which considerably decreases number of necessary initial data of theory of field E_σ.

The most reliable pieces of information about lower layers of ionosphere are the following.

1. Experimental data on absorption and reflection of long, medium, and short waves, indicating existence in daytime of ionospheric layers below E layer and absence of layers of ionization below E layer at night [1-5].

2. Theoretical research [38, 39] on microprocesses in lower layers of ionosphere, from which it follows that at heights of 70-90 km basic role on frequencies of 10-20 kilocycles is played by electrons.

3. Theoretical treatment of different experiments [40], allowing us to estimate dependence of effective number of collisions ν_{eff} electrons with other particles on height of layers h in interval of 60-100 km (Fig. 2).

4. Experimental data on magnitude of magnetic field of earth H_0.

From first point above the conclusion can be made that very long waves are reflected from ionospheric layers located at heights of 70-90 km, which it is possible to approximate by concentric (model B) or approximately concentric (model C) layers ($\mathbf{r}_1$, $\mathbf{r}_2$), ($\mathbf{r}_2$, $\mathbf{r}_3$) ... ($\mathbf{r}_{N-1}$, ∞), where $(\mathbf{r}_1 - \mathbf{r}_0)$ = 70 km and $(\mathbf{r}_{N-1} - \mathbf{r}_0)$ = 100 km.

Thus the last layer of model of ionosphere ($\mathbf{r}_{N-1}$, ∞) is propagated infinitum. Distinction between model of medium from real ionosphere in region $\mathbf{r} > \mathbf{r}_{N-1}$ is immaterial, since VLW's do not penetrate of E layer.

Another important point for accurate representation of model of medium is electron character of conductivity of lower layers of ionosphere. Considering that motion of electrons under action of harmonic field $Ee^{-i\omega t}$ in lower layer of ionosphere

is described by Lorentz equation

$$-i\omega m\bar{v} + \nu_{eff} m\bar{v} = e\bar{E} + \frac{e}{c}[\bar{v}, H_0],$$

where ω – angular frequency of electromagnetic field,

m – mass and e – charge of electron,

$\bar{v}$ – velocity vector of electron,

we obtain, disregarding horizontal component $\bar{H}_0$, following tensor of dielectric constant for k layer:

$$\|\bar{\varepsilon}_k\| = \begin{Vmatrix} \varepsilon^k_{rr}, & 0, & 0 \\ 0, & \varepsilon^k_{\theta\theta}, & \varepsilon^k_{\theta\varphi} \\ 0, & \varepsilon^k_{\varphi\theta}, & \varepsilon^k_{\varphi\varphi} \end{Vmatrix} \tag{2.2}$$

for $r_{k-1} < r < r_k$ $(k = 2, 3, 4 \ldots N)$,

where

$$\varepsilon_{rr} = 1 + \frac{\omega_0^2 i}{\omega(\nu_{eff} - i\omega)}, \tag{2.3}$$

$$\varepsilon_{\theta\theta} = \varepsilon_{\varphi\varphi} = 1 + \frac{i\omega_0^2(\nu_{eff} - i\omega)}{\omega[(\nu_{eff} - i\omega)^2 + \omega_H^2]}, \tag{2.4}$$

$$\varepsilon_{\theta\varphi} = -\varepsilon_{\varphi\theta} = -\frac{i\omega_0^2\omega_H}{\omega[(\nu_{eff} - i\omega)^2 + \omega_H^2]}. \tag{2.5}$$

In formulas (2.3)-(2.5) are introduced following conventional designations:

$$\omega_0^2 = \frac{4\pi N_e e^2}{m} \text{ — square of critical frequency,} \tag{2.6}$$

$$\omega_H = -\frac{eH_0}{mc} \text{ — gyromagnetic frequency.} \tag{2.7}$$

Index of k layer in formulas (2.3)-(2.7) is omitted. Thus every "k" layer of ionosphere is characterized by four quantities ω_0^2, k, ν_{eff}, k, ω_H, k, and r_k.

Quantity ν_{eff}, k is connected with height of ionosphere $h = r_k - r_0$, according to work Nicolet [40], by formulas

$$\nu_{eff} = 5{,}22.10^{9}\frac{p}{\sqrt{T}} \text{ for } h < 90 \text{ км},$$

$$\nu_{eff} = 3{,}74.10^{9}\frac{p}{\sqrt{T}} \text{ for } h > 80 \text{ км},$$

which are depicted in Fig. 2.

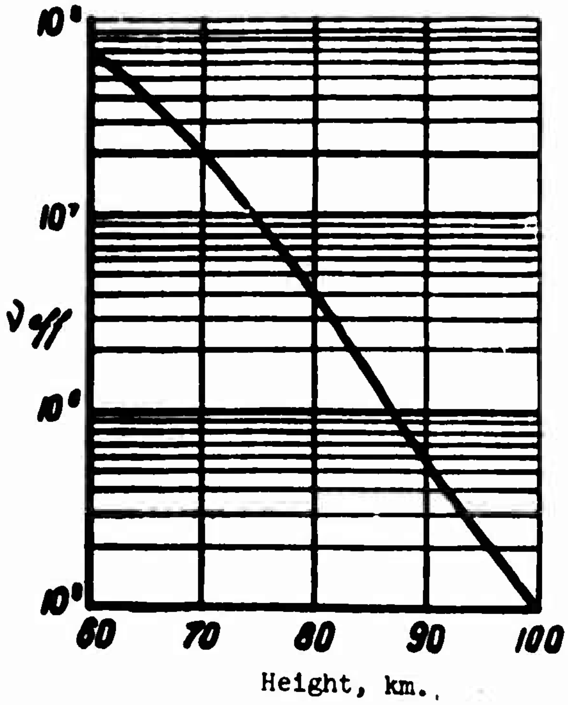


Fig. 2.

Quantity ω_H is calculated from experimental value of $\overline{H}_0$ and is approximately equal to 8.10^6 for all layers in interval of 70-90 km at mid and northern latitudes. Thus unknown quantities in models B and C are distribution of electron concentration, determined by quantities $\omega^2_{0,k}$ and r_k. Further specification of models B and C on the basis of known data on medium is not possible, and parameters ω^2_{0k} and r_k, which we designated in Chapter I through x, y, z, ..., are determined from solution of mixed problem per diagram of Fig. 1.

§ 2. Formulation of Boundary Problem on Basis of Model B

In accordance with method of solution of mixed problem presented in § 1, Chapter I we first solve primal problem of theory of VLW propagation on the basis of model B, considering that all parameters of model B are known. Primal problem of VLW propagation on the basis of model B is formulated in the following way.

In medium there exist distributed outside currents, harmonic in time and directed radially

$$I_{стор} = I_{стор}(r,\theta)\, e^{-i\omega t} = I_r(r,\theta)\, e^{-i\omega t}. \tag{2.8}$$

It is assumed that distribution function of outside currents $I_r(r, \theta)$ depends only on spherical coordinates r and θ, that is, is characterized by symmetry of rotation with respect to polar axis $\theta = 0$ of spherical system of coordinates r, θ, φ.

To be found is stationary electromagnetic field created by currents (2.8) in spherically laminar medium described by following tensors of dielectric constant

$$\|\varepsilon_k\| = \begin{Vmatrix} \varepsilon^k_{rr}, & 0, & 0 \\ 0, & \varepsilon^k_{\theta\theta}, & \varepsilon^k_{\theta\varphi} \\ 0, & -\varepsilon^k_{\theta\varphi}, & \varepsilon^k_{\theta\theta} \end{Vmatrix} \tag{2.9}$$

$$r_{k-1} < r < r_k;\quad k = 0, 1, 2, 3 \dots N,$$

where

for zero layer ($k = 0$); ($r_{-1} = 0$, r_0)

$$\varepsilon^0_{rr} = \varepsilon^0_{\theta\theta} = \varepsilon_0' + i\,\frac{4\pi\sigma_0}{\omega};\ \varepsilon^0_{\theta\varphi} = 0; \tag{2.10}$$

for first layer ($k = 1$); ($r_0 = a$, r_1)

$$\varepsilon^1_{rr} = \varepsilon^1_{\theta\theta} = 1; \quad \varepsilon^1_{\theta\varphi} = 0; \tag{2.11}$$

and for other $N - 1$ layers ($k = 2, 3, \dots N$)

$$(r_1, r_2); (r_2, r_3) \dots (r_{N-1}, r_N = \infty) \tag{2.12}$$

tensor $\|\varepsilon_k\|$ is described by formulas (2.3)-(2.7).

In virtue of symmetry of excitation currents (2.8), electromagnetic field should be determined from Maxwell equations, in which all derivatives with respect to φ are equal to zero. Thus for every spherical "k" layer components of field E_φ, E_θ, E_r, and H_φ, H_θ, H_r are determined by following system of Maxwell equations, recorded in spherical system of coordinates (CGS):

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta \cdot H^k_\varphi) = -\frac{i\omega}{c}\varepsilon^k_{rr}E^k_r + \frac{4\pi}{c}I^k_r, \tag{2.13}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH^k_\varphi) = \frac{i\omega}{c}[\varepsilon^k_{\theta\theta}E^k_\theta + \varepsilon^k_{\theta\varphi}E^k_\varphi], \tag{2.14}$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}(rH^k_\theta) - \frac{\partial H^k_r}{\partial\theta}\right] = -\frac{i\omega}{c}[\varepsilon^k_{\varphi\theta}E^k_\theta + \varepsilon^k_{\varphi\varphi}E^k_\varphi]. \tag{2.15}$$

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta \cdot E^k_\varphi) = \frac{i\omega}{c}H^k_r, \tag{2.16}$$

$$-\frac{1}{r}\frac{\partial}{\partial r}(rE^k_\varphi) = \frac{i\omega}{c}H^k_\theta, \tag{2.17}$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}(rE^k_\theta) - \frac{\partial E^k_r}{\partial\theta}\right] = \frac{i\omega}{c}H^k_\varphi \tag{2.18}$$

in interval $r_{k-1} \leq r \leq r_k$; $k = 0, 1, 2, 3, \dots N$.

On boundaries of layers have to be met conditions of continuity of tangential components:

$$E^k_\varphi|_{r_k} = E^{k+1}_\varphi|_{r_k}, \tag{2.19}$$

$$E^k_\theta|_{r_k} = E^{k+1}_\theta|_{r_k}, \tag{2.20}$$

$$H^k_\varphi|_{r_k} = H^{k+1}_\varphi|_{r_k}, \tag{2.21}$$

$$H^k_\theta|_{r_k} = H^{k+1}_\theta|_{r_k}, \tag{2.22}$$

where $k = 0, 1, 2, \dots N - 1$.

Besides, there have to be met conditions of limitedness of solutions at point $r_{-1} = 0$

$$\mathrm{mod}\, E|_{r=1} < M; \quad \mathrm{mod}\, H|_{r=1} < M \tag{2.23}$$

and condition at infinity

$$\mathrm{mod}\, E|_{r\to\infty} \to 0; \quad \mathrm{mod}\, H|_{r\to\infty} \to 0. \tag{2.24}$$

Equations (2.13)-(2.18) under conditions on boundaries of layers (2.19)-(2.22) and conditions at singular points (2.23), (2.24) for finite values $\nu_{eff,N}$ and $N_{e,N}$ in simple form determine electromagnetic field ($\bar{E}$, $\bar{H}$) for given I_{CTOP}.

§ 3. Introduction of Electrical and Magnetic Potentials A and B of Potential Vector Function $\begin{vmatrix} B \\ A \end{vmatrix}$

For solution of boundary problem formulated at the end of preceding paragraph we introduce two scalar functions A and B, determined through components E_φ and H_φ:

$$E_\varphi = \frac{1}{r}\frac{\partial A}{\partial \theta}, \tag{2.25}$$

$$H_\varphi = \frac{1}{r}\frac{\partial B}{\partial \theta}. \tag{2.26}$$

Then, according to (2.13)-(2.18), all remaining components are determined through A and B in the following way:

$$E_r = \frac{i}{k_1 \varepsilon_{rr} r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial B}{\partial \theta}\right), \tag{2.27}$$

$$E_\theta = -\frac{i}{k_1 \varepsilon_{\theta\theta} r}\frac{\partial^2 B}{\partial r \partial \theta} - \frac{\varepsilon_{\theta\varphi}}{r \varepsilon_{\theta\theta}}\frac{\partial A}{\partial \theta}, \tag{2.28}$$

$$H_r = -\frac{i}{k_1 r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial A}{\partial \theta}\right), \tag{2.29}$$

$$H_\theta = \frac{i}{k_1 r}\frac{\partial^2 A}{\partial r \partial \theta}, \tag{2.30}$$

where $k_1 = \frac{\omega}{c}$ is wave number of free space.

After introduction of functions A and B, system of Maxwell equations (2.13)-(2.18) takes form of series of N + 1 pairs of connected scalar wave equations of second order:

$$\frac{\varepsilon^k_{rr}}{\varepsilon^k_{\theta\theta}}\frac{\partial^2 B_k}{\partial r^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial B_k}{\partial \theta}\right) + k_1^2 \varepsilon^k_{rr} B_k - i k_1 \varepsilon^k_{rr}\frac{\varepsilon^k_{\theta\varphi}}{\varepsilon^k_{\theta\theta}}\frac{\partial A_k}{\partial r} = \frac{4\pi}{c} I_k, \tag{2.31}$$

$$\frac{\partial^2 A_k}{\partial r^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial A_k}{\partial \theta}\right) + k_1^2\left[\varepsilon^k_{\theta\theta} + \frac{(\varepsilon^k_{\theta\varphi})^2}{\varepsilon^k_{\theta\theta}}\right] A_k + i k_1 \frac{\varepsilon^k_{\theta\varphi}}{\varepsilon^k_{\theta\theta}}\frac{\partial B_k}{\partial r} = 0 \tag{2.32}$$

in interval $r_{k-1} \leq r \leq r_k$ k = 0, 1, 2, 3, ... N.

System of equations (2.31) and (2.32), according to (2.19)-(2.22), should satisfy conditions of jumps for A and B on internal boundaries of media

$$A_k\Big|_{r_k} = A_{k+1}\Big|_{r_k}, \tag{2.33}$$

$$B_k\Big|_{r_k} = B_{k+1}\Big|_{r_k}, \tag{2.34}$$

$$\frac{\partial A_k}{\partial r}\Big|_{r_k} = \frac{\partial A_{k+1}}{\partial r}\Big|_{r_k}, \tag{2.35}$$

$$\frac{1}{\varepsilon^k_{\theta\theta}}\frac{\partial B_k}{\partial r}\Big|_{r_k} - \frac{ik_1\varepsilon^k_{\theta\varphi}}{\varepsilon^k_{\theta\theta}}A_k\Big|_{r_k} = \frac{1}{\varepsilon^{k+1}_{\theta\theta}}\frac{\partial B_{k+1}}{\partial r}\Big|_{r_k} - \frac{ik_1\varepsilon^{(k+1)}_{\theta\varphi}}{\varepsilon^{(k+1)}_{\theta\theta}}A_{k+1}\Big|_{r_k}, \tag{2.36}$$

where k = 0, 1, 2, 3, ... N - 1,

and also, according to (2.23) and (2.24), boundary conditions:

$$\mathrm{mod}\, A_0 < M';\ \mathrm{mod}\, B_0 < N' \text{ for } r \to 0, \tag{2.37}$$

where M' and N' are finite real numbers

$$\mathrm{mod}\, A \to 0;\ \ \mathrm{mod}\, B \to 0 \text{ for } r \to \infty. \tag{2.38}$$

Now we introduce vector function of potentials with components A and B $\begin{vmatrix} B \\ A \end{vmatrix}$ and vector function of currents $\begin{vmatrix} I \\ 0 \end{vmatrix}$. Then we record system of equations (2.31)-(2.32) in operator differential-matrix form

$$l_r^{(k)}\begin{vmatrix} B_k \\ A_k \end{vmatrix} + l_\theta^{(k)}\begin{vmatrix} B_k \\ A_k \end{vmatrix} = \frac{4\pi}{c}r^2\begin{vmatrix} I \\ 0 \end{vmatrix}, \tag{2.39}$$

where $l_r^{(k)}$ is matrix operator

$$l_r^{(k)} = \begin{vmatrix} \dfrac{\varepsilon^k_{rr}}{\varepsilon^k_{\theta\theta}} r^2 \dfrac{\partial^2}{\partial r^2}\cdot + k_1^2\varepsilon^k_{rr} r^2 \cdot & -\dfrac{ik_1\varepsilon^k_{rr}\varepsilon^k_{\theta\varphi} r^2}{\varepsilon^k_{\theta\theta}}\dfrac{\partial}{\partial r}\cdot \\ \dfrac{ik_1 r^2\varepsilon^k_{\theta\varphi}}{\varepsilon^k_{\theta\theta}}\dfrac{\partial}{\partial r}\cdot & r^2\dfrac{\partial^2}{\partial r^2}\cdot + k_1^2 r^2\left[\varepsilon^k_{\theta\theta} + \dfrac{(\varepsilon^k_{\theta\varphi})^2}{\varepsilon^k_{\theta\theta}}\right]\cdot \end{vmatrix} \tag{2.40}$$

in interval (r_{k-1}, r_k) k = 0, 1, 2, ...N,

and $l_\theta^{(k)}$ is matrix operator

$$l_\theta^k = l_\theta = \begin{vmatrix} \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\cdot & 0 \\ 0 & \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\cdot \end{vmatrix} \qquad (2.41)$$

Corresponding conditions of jump take form:

$$\begin{vmatrix} \frac{1}{\varepsilon_{\theta\theta}^k}\frac{\partial}{\partial r}\cdot & -\frac{ik_1\varepsilon_{\theta\varphi}^k}{\varepsilon_{\theta\theta}^k}\cdot \\ 0 & \frac{\partial}{\partial r} \end{vmatrix} \times \begin{vmatrix} B_k \\ A_k \end{vmatrix}_{r_k} = \begin{vmatrix} \frac{1}{\varepsilon_{\theta\theta}^{k+1}}\frac{\partial}{\partial r}\cdot & -\frac{ik_1\varepsilon_{\theta\varphi}^{(k+1)}}{\varepsilon_{\theta\theta}^{(k+1)}} \\ 0 & \frac{\partial}{\partial r}\cdot \end{vmatrix} \times \begin{vmatrix} B_{k+1} \\ A_{k+1} \end{vmatrix}_{r_k} \qquad (2.42)$$

$$\begin{vmatrix} B_k \\ A_k \end{vmatrix}_{r_k} = \begin{vmatrix} B_{k+1} \\ A_{k+1} \end{vmatrix}_{r_k} \qquad (2.43)$$

$$k = 0, 1, 2 \; .... \; N-1,$$

and boundary conditions (2.37) and (2.38):

$$\mathrm{mod} \begin{vmatrix} B \\ A \end{vmatrix} < \begin{vmatrix} M' \\ N' \end{vmatrix} \text{ when } r \to 0, \qquad (2.44)$$

$$\mathrm{mod} \begin{vmatrix} B \\ A \end{vmatrix} \to 0 \quad \text{when} \to \infty. \qquad (2.45)$$

Since differential-matrix operators $l_r^{(k)}$ determined by (2.40) differ from each other only in numerical values of parameters $\varepsilon_{rr}^{(k)}$, $\varepsilon_{\theta\theta}^k$, and $\varepsilon_{\theta\varphi}^k$, we shall consider system of equations (2.39) k = 0, 1, 2, ... N for intervals (r_{k-1}, r_k), k = 0, 1, 2, ... N as one equation in interval $(0, \infty)$

$$l_r \begin{vmatrix} B \\ A \end{vmatrix} + l_\theta \begin{vmatrix} B \\ A \end{vmatrix} = \frac{4\pi}{c} \begin{vmatrix} I \\ 0 \end{vmatrix}, \qquad (2.46)$$

in which operator l_r consecutively takes forms $l_r^{(0)}$, $l_r^{(1)}$, $l_r^{(2)}$, ... $l_r^{(N)}$ in accordance with passage through varying r intervals $(0, r_0)$, (r_0, r_1), (r_1, r_2) (r_1, r_2) ... (r_{N-1}, ∞). Vector function $\begin{vmatrix} B \\ A \end{vmatrix}$, taking values $\begin{vmatrix} B_k \\ A_k \end{vmatrix}$, k = 0, 1, 2, ... N, in corresponding intervals $(0, r_0)$, (r_0, r_1) ... (r_{N-1}, ∞) undergoes breaks at points r_0, r_1, r_2, ... r_{N-1}, where it satisfies conditions of break (2.42)-(2.43).

For N + 1 layers $(0, r_0)$, (r_0, r_1), ... (r_{N-1}, ∞) there are 2N conditions of break (2.42) and (2.43) and two conditions (2.44) and (2.45) at boundaries of interval $(0, \infty)$, 2(N + 1) conditions in all, determining 2(N + 1) arbitrary constant solutions of equation (2.46).

Operator l_0 has two singular points $\theta = 0$ and $\theta = \pi$ at which we shall require limitedness of solution $\begin{vmatrix} B \\ A \end{vmatrix}$ of equation (2.46).

§ 4. Operator of Normal Waves and Its Spectrum. Free Normal Waves

For finding solution of nonuniform equation (2.46) under conditions (2.42)-(2.45) we apply method of normal waves presented in works [36, 37]. According to method of normal waves, we first find particular solutions of homogeneous equation (2.46) for conditions (2.42)-(2.45), having form

$$l_r\begin{vmatrix} B \\ A \end{vmatrix} + l_0\begin{vmatrix} B \\ A \end{vmatrix} = 0. \tag{2.47}$$

Let us find particular solution of (2.47) in the form

$$\begin{vmatrix} B \\ A \end{vmatrix} = \begin{vmatrix} Y(r) \\ Z(r) \end{vmatrix} \cdot \Psi(\theta), \tag{2.48}$$

where

$$\begin{vmatrix} Y(r) \\ Z(r) \end{vmatrix} = \begin{vmatrix} Y_k(r) \\ Z_k(r) \end{vmatrix} \quad \text{in interval } r_{k-1} < r < r_k \tag{2.49}$$

Putting (2.48) in (2.47) and dividing variables, we obtain

$$\frac{l_r\begin{vmatrix} Y(r) \\ Z(r) \end{vmatrix}}{\begin{vmatrix} Y(r) \\ Z(r) \end{vmatrix}} = -\frac{l_0\Psi(\theta)}{\Psi(\theta)}. \tag{2.50}$$

Identity (2.50) can exist only under the condition that both its parts are equal to the same constant, which we designate by letter χ. Then we obtain two equations:

$$l_r\begin{vmatrix} Y(r) \\ Z(r) \end{vmatrix} = \chi\begin{vmatrix} Y(r) \\ Z(r) \end{vmatrix}. \tag{2.51}$$

$$l_0\Psi(\theta) + \chi\Psi(\theta) = 0. \tag{2.52}$$

First equation in expanded form will be

$$\begin{vmatrix} \frac{\varepsilon_{rr}}{\varepsilon_{\theta\theta}} r^2\frac{d^2}{dr^2}\cdot + k_1^2\varepsilon_{rr}r^2. & -\frac{ik_1\varepsilon_{rr}\varepsilon_{\theta\varphi}}{\varepsilon_{\theta\theta}} r^2\frac{d}{dr}. \\ \frac{ik_1\varepsilon_{\theta\varphi}}{\varepsilon_{\theta\theta}} r^2\frac{d}{dr}. & r^2\frac{d^2}{dr^2}\cdot + k_1^2\left(\varepsilon_{\theta\theta} + \frac{\varepsilon_{\theta\varphi}^2}{\varepsilon_{\theta\theta}}\right)r^2. \end{vmatrix} \times \begin{vmatrix} Y \\ Z \end{vmatrix} = \chi\cdot\begin{vmatrix} Y \\ Z \end{vmatrix}. \tag{2.53}$$

where $\varepsilon_{rr} = \varepsilon_{rr}^k$; $\varepsilon_{\theta\theta} = \varepsilon_{\theta\theta}^k$; $\varepsilon_{\theta\varphi} = \varepsilon_{\theta\varphi}^k$ in intervals (r_{k-1}, r_k)

for $k = 0, 1, 2, 3, \ldots N$.

On boundaries of intervals r_k; k = 0, 1, 2, 3, ... N - 1 conditions of jump are met:

$$\left\| \begin{matrix} \frac{1}{\varepsilon_{\theta\theta}^{k}} \frac{d}{dr} \cdot & -\frac{ik_1 \varepsilon_{\theta\varphi}^{k}}{\varepsilon_{\theta\theta}^{k}} \cdot \\ 0 & \frac{d}{dr} \end{matrix} \right\| \times \left\| \begin{matrix} Y_k \\ Z_k \end{matrix} \right\|_{r_k} =$$

$$= \left\| \begin{matrix} \frac{1}{\varepsilon_{\theta\theta}^{(k+1)}} \frac{d}{dr} \cdot & -\frac{ik_1 \varepsilon_{\theta\varphi}^{(k+1)}}{\varepsilon_{\theta\theta}^{(k+1)}} \cdot \\ 0 & \frac{d}{dr} \cdot \end{matrix} \right\| \times \left\| \begin{matrix} Y_{k+1} \\ Z_{k+1} \end{matrix} \right\|_{r_k}, \tag{2.54}$$

$$\left\| \begin{matrix} Y_k \\ Z_k \end{matrix} \right\|_{r_k} = \left\| \begin{matrix} Y_{k+1} \\ Z_{k+1} \end{matrix} \right\|_{r_k} \tag{2.55}$$

and on ends of zero and N^{th} intervals boundary conditions

$$\operatorname{mod} \left| \begin{matrix} Y \\ Z \end{matrix} \right|_{r \to 0} \to \text{limit} \tag{2.56}$$

$$\operatorname{mod} \left| \begin{matrix} Y \\ Z \end{matrix} \right|_{r \to \infty} \to 0. \tag{2.57}$$

Differential-matrix expression presented in (2.53) for boundary conditions (2.54), (2.55), (2.56), and (2.57) is operator of normal waves, which acts on vector function [**Y/Z**]. Designating it by symbol $\mathbf{L}_r$, we record equation (2.53) with conditions (2.54)-(2.57) in the form

$$L_r \left| \begin{matrix} Y \\ Z \end{matrix} \right| = \chi \left| \begin{matrix} Y \\ Z \end{matrix} \right|. \tag{2.58}$$

Equation (2.58) determines spectrum of operator of normal waves. In general operator $\mathbf{L}_r$ is not self-conjugate and singular. Its spectrum consists of discrete and continuous parts. In case of great conductivity of earth $|\varepsilon_0| \gg 1$, solid part of spectrum may be disregarded in examining field in interval ($\mathbf{r}_0$, ∞) and under the condition that exciting currents are located outside earth. Therefore we shall consider subsequently only discrete part of spectrum, that is, eigenvalues of operator $\mathbf{L}_r$. Let us renumber them in order of growth of absolute values

$$\chi_0, \chi_1, \chi_2, \chi_3 \ldots \chi_j. \tag{2.59}$$

Since operator $\mathbf{L}_r$ is not self-conjugate, eigenvalues of χ_j will be complex

$$\chi_j = Re(\chi_j) + i\,Im(\chi_j) \quad j = 0, 1, 2 \ldots \tag{2.59'}$$

From what is to follow one will see that to every eigenvalue of χ_j there corresponds only one eigenvector function

$$\begin{Vmatrix} Y_j(r) \\ Z_j(r) \end{Vmatrix} = \begin{Vmatrix} Y_{j,k}(r) \\ Z_{j,k}(r) \end{Vmatrix} \tag{2.60}$$

for $r_{k-1} \leq r \leq r_k$ $k = 0, 1, 2, \ldots N.$

Eigenvector functions (2.60) of operator L_r are orthogonal eigenfunctions of conjugate operator L_r^*, which is determined from Lagrange identity

$$\left(L_r \begin{Vmatrix} Y \\ Z \end{Vmatrix}, \begin{Vmatrix} U \\ V \end{Vmatrix}\right) = \left(\begin{Vmatrix} Y \\ Z \end{Vmatrix}, L_r^* \begin{Vmatrix} U \\ V \end{Vmatrix}\right), \tag{2.61}$$

where parentheses designate scalar product of two vector functions.

Let us designate eigenvalues and vector functions of conjugate operator respectively

$$\mu_0, \mu_1, \mu_2 \cdots \mu_p \cdots$$

$$\begin{Vmatrix} U_0 \\ V_0 \end{Vmatrix}; \begin{Vmatrix} U_1 \\ V_1 \end{Vmatrix}; \begin{Vmatrix} U_2 \\ V_2 \end{Vmatrix} \cdots \begin{Vmatrix} U_p \\ V_p \end{Vmatrix} \cdots, \tag{2.62}$$

which satisfy equation

$$L_r^* \begin{Vmatrix} U \\ V \end{Vmatrix} = \mu \begin{Vmatrix} U \\ V \end{Vmatrix} \tag{2.63}$$

From (2.58) and (2.63) under condition (2.61) follows relationship

$$\left(\begin{Vmatrix} Y_j \\ Z_j \end{Vmatrix}, \begin{Vmatrix} U_p \\ V_p \end{Vmatrix}\right) = 0 \text{ for } j \neq p \tag{2.64}$$

or in scanned form

$$\int_0^\infty Y_j(r)\, U_p(r)\, dr + \int_0^\infty Z_j(r) \cdot V_p(r)\, dr = 0; \quad j \neq p, \tag{2.64'}$$

expressing orthogonality of eigenfunctions of conjugate operators L_r and L_r^*.

For operator L_r, generated by expression (2.53) and boundary conditions (2.54)-(2.57), it is easy to show that

$$\mu_p = \bar{\lambda}_p, \tag{2.65}$$

$$\begin{Vmatrix} U_p \\ V_p \end{Vmatrix} = \text{const.} \begin{Vmatrix} \bar{Y}_p \sqrt{\varepsilon_r}\, r^2 \\ \bar{Z}_p / r^2 \end{Vmatrix}, \tag{2.66}$$

where vinculum denotes conjugate complex numbers.

In virtue of (2.65) and (2.66), condition of orthogonality for non-self-conjugate operator L_r takes form

$$\left(\begin{Vmatrix} Y_j \\ Z_j \end{Vmatrix}, \begin{Vmatrix} \bar{Y}_p \sqrt{\varepsilon_r}\, r^2 \\ \bar{Z}_p / r^2 \end{Vmatrix}\right) = 0 \quad j \neq p \tag{2.67}$$

or in expanded form

$$\int_0^\infty \frac{Y_j Y_p}{\varepsilon_w r^2} dr + \int_0^\infty \frac{Z_j Z_p}{r^2} dr = 0 \quad j \neq p. \tag{2.68}$$

If j = p, then (2.64) is equal to N_j, called subsequently the normalizing factor

$$N_j = \int_0^\infty \frac{Y_j^2 dr}{\varepsilon_w r^2} + \int_0^\infty \frac{Z_j^2}{r^2} dr. \tag{2.69}$$

Now let us return to second equation (2.52), which for eigenvalue of χ_j in expanded form has the form

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Psi_j}{d\theta}\right) + \chi_j \Psi_j = 0. \tag{2.52'}$$

Introducing new designation

$$\chi_j = \nu_j(\nu_j + 1), \tag{2.70}$$

we convert it to Legendre's equation, which has two particular solutions [37]:

$$\Psi_j^{(1)} = L_{\nu_j}^{(1)}(\cos\theta) = Q_{\nu_j} + i\frac{\pi}{2}P_{\nu_j}. \tag{2.71}$$

$$\Psi_j^{(2)} = L_{\nu_j}^{(2)}(\cos\theta) = Q_{\nu_j} - i\frac{\pi}{2}P_{\nu_j}. \tag{2.72}$$

Thus particular solution (2.48) of equation corresponding to eigenvalue of χ_j has the form

$$\begin{vmatrix} B_j \\ A_j \end{vmatrix} e^{-i\omega t} = \begin{vmatrix} Y_j(r) \\ Z_j(r) \end{vmatrix} \cdot \left\{ C_1 L_{\nu_j}^{(1)} + C_2 L_{\nu_j}^{(2)} \right\} e^{-i\omega t}. \tag{2.73}$$

Here we introduced time factor $e^{-i\omega t}$. In region of permissibility of asymptotic representations $L_\nu^{(1)}$ and $L_\nu^{(2)}$ [37] we record (2.73) in the form

$$\begin{vmatrix} Y_j(r) \\ Z_j(r) \end{vmatrix} \cdot \sqrt{\frac{\pi}{2\nu_j \sin\theta}} \left\{ C_1 e^{i\left[\nu_j'\theta + \frac{\pi}{4}\right]} + C_2 e^{-i\left[\nu_j'\theta + \frac{\pi}{4}\right]} \right\} e^{-i\omega t}, \tag{2.74}$$

where

$$\nu_j' = \nu_j + \frac{1}{2} = \sqrt{+\chi_j + \frac{1}{4}} \approx \sqrt{+\chi_j}. \tag{2.75}$$

(2.74) represents sum of two waves traveling in positive and negative directions on coordinate θ with angular phase velocity determined by angular wave number ν_j. Both waves are modulated in identical manner with respect to front of wave θ = const

according to eigenfunction $|Y_j(r)|Z_j(r)|$. Such waves were called normal, since they were obtained as a result of solution of problem for eigenvalues of operator of normal waves, just as normal oscillations are obtained as a result of solution of problem for eigenvalues of operator of normal oscillations.

Sum of normal waves (2.73) represents any wave process harmonic in time in region where exciting currents $\mathbf{I}_{\text{стор}}$ are absent. Therefore by analogy with normal oscillations, such waves are called free normal waves [36, 37].

Solution of equation (2.46) on right side yields forced normal waves, which wi be considered in the following paragraph.

§ 5. Formulation of Solution of Nonuniform Boundary Problem. Forced Normal Waves

Now, following method of normal waves, let us look for solution of nonuniform equation (2.46)

$$L_r\begin{vmatrix} B \\ A \end{vmatrix} + L_\theta\begin{vmatrix} B \\ A \end{vmatrix} = \frac{4\pi}{c} r^2 \begin{vmatrix} I_r \\ 0 \end{vmatrix} \tag{2.76}$$

in the form of spectrum of normal waves

$$\begin{vmatrix} B \\ A \end{vmatrix} = \sum_{j=0}^{\infty}\begin{vmatrix} Y_j(r) \\ Z_j(r) \end{vmatrix} \Phi_j(\theta). \tag{2.77}$$

We recall that $\mathbf{L}_r$ is operator with respect to coordinate r, generated by differential matrix expression (2.53) and boundary conditions (2.54)-(2.57), while $\mathbf{L}_\theta$ is operator with respect coordinate θ, generated by differential expression (2.41) and conditions of limitedness of solution at points $\theta = 0$ and $\theta = \pi$.

For determination of unknown phase factors we put (2.77) in (2.76)

$$L_r\sum_{j=0}^{\infty}\begin{vmatrix} Y_j \\ Z_j \end{vmatrix}\Phi_j + L_\theta\sum_{j=0}^{\infty}\begin{vmatrix} Y_j \\ Z_j \end{vmatrix}\Phi_j = \frac{4\pi r^2}{c}\begin{vmatrix} I_r \\ 0 \end{vmatrix}. \tag{2.78}$$

Sinc series (2.77) converge evenly, we change order of summation and operations $\mathbf{L}_r$ and $\mathbf{L}_\theta$

$$\sum_{j=0}^{\infty}\Phi_j(\theta)\cdot L_r\begin{vmatrix} Y_j \\ Z_j \end{vmatrix} + \sum_{j=0}^{\infty}\begin{vmatrix} Y_j \\ Z_j \end{vmatrix} L_\theta\cdot\Phi_j = \frac{4\pi}{c} r^2\begin{vmatrix} I_r \\ 0 \end{vmatrix}. \tag{2.79}$$

Multiplying both parts of (2.79) scalarly on some eigenvector of conjugate operator $\mathbf{L}_r{}^*$ and taking into account (2.58), we obtain

$$-\sum_{j=0}^{\infty}\Phi_j\chi_j\left(\begin{vmatrix} Y_j \\ Z_j \end{vmatrix}, \begin{vmatrix} U_p \\ V_p \end{vmatrix}\right) + \sum_{j=0}^{\infty}\left(\begin{vmatrix} Y_j \\ Z_j \end{vmatrix}, \begin{vmatrix} U_p \\ V_p \end{vmatrix}\right) L_\theta\cdot\Phi_j = \frac{4\pi}{c}\left(\begin{vmatrix} I_r \\ 0 \end{vmatrix}, \begin{vmatrix} r^2U_p \\ r^2V_p \end{vmatrix}\right). \tag{2.80}$$

In virtue of condition of orthogonality of (2.64), expression (2.80) is simplified

$$L_\theta \Phi_j + \chi_j \Phi_j = \frac{4\pi}{cN_p}\left(\left\| \begin{matrix} I_r \\ 0 \end{matrix} \right\|, \left\| \begin{matrix} r^2 U_p \\ r^2 V_p \end{matrix} \right\|\right). \qquad (2.80')$$

where N_p is normalizing factor (2.69).

Expanding operation $\mathbf{L}_\theta$ and using (2.66), we obtain equation

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Phi_j}{d\theta}\right) + \chi_j \Phi_j = \frac{4\pi}{cN_j}\int_0^\infty \frac{I_r Y_j}{\varepsilon_{rr}}\,dr, \qquad (2.81)$$

which, under condition of limitedness of solution at points $\theta = 0$ and $\theta = \pi$, determines phase factors $\Phi_j(\theta)$ of normal waves. These factors differ from phase factors $\Psi_j(\theta)$ of free normal waves, considered in § 4, Chapter II, since they are forced solutions of Legendre equation on right side (2.81).

Forced solution of equation (2.81) can be found by different methods. In work [37] it was obtained with help of Green's function. Here we shall solve equation (2.81) by method of variation of arbitrary constants of general solution of homogeneous Legendre equation, which it is possible to represent in the form

$$A_j P_{\nu_j}[\cos(\pi - \theta)] + B_j P_{\nu_j}(\cos\theta). \qquad (2.82)$$

Considering $\mathbf{A}_j$ and $\mathbf{B}_j$ functions of θ, we find them from two following equations:

$$A_j'(\xi)P_{\nu_j}(-\xi) + B_j'(\xi)P_{\nu_j}(\xi) = 0, \qquad (2.83)$$

$$A_j'(\xi)P_{\nu_j}(-\xi) + B_j'(\xi)P_{\nu_j}(\xi) = \frac{4\pi}{cN_j} I_j, \qquad (2.84)$$

where primes denote derivatives with respect to θ, and

$$\xi = \cos\theta, \qquad (2.85)$$

$$I_j = \int_0^\infty \frac{I_r Y_j(r)\,dr}{\varepsilon_{rr}}. \qquad (2.86)$$

From these equations it follows that:

$$A_j(\xi) = -\frac{4\pi}{cN_j}\int_0^\theta \frac{I_j \cdot P_{\nu_j}(\xi)\,d\theta}{\Delta_j}, \qquad (2.87)$$

$$B_j(\xi) = \frac{4\pi}{cN_j}\int_0^\theta \frac{I_j \cdot P_{\nu_j}(-\xi)\,d\theta}{\Delta_j},$$

where

$$\Delta_j = \frac{2}{\pi}\frac{\sin\nu_j\pi}{\sin\theta}. \qquad (\;.\;)$$

Thus particular solution of nonuniform Legendre equation (2.81) has the form

$$-P_{\nu_j}(-\xi)\frac{4\pi}{cN_j}\int\limits_0^\theta \frac{I_j P_{\nu_j}(\xi)\,d\theta}{\Delta_j}+P_{\nu_j}(\xi)\frac{4\pi}{cN_j}\int\limits_0^\theta \frac{I_j P_{\nu_j}(-\xi)\,d\theta}{\Delta_j},$$

and general solution, after substitution of (2.88),

$$\Phi_j(\xi)=C_jP_{\nu_j}[\cos(\pi-\theta)]+D_j\,P_{\nu_j}[\cos\theta]-$$

$$-\frac{2\pi^2}{cN_j\sin\nu_j\pi}P_{\nu_j}[\cos(\pi-\theta)]\cdot\int\limits_0^\theta I_j(\theta')P_{\nu_j}(\cos\theta')\sin\theta'\,d\theta'+$$

$$+\frac{2\pi^2}{cN_j\sin\nu_j\pi}P_{\nu_j}[\cos\theta]\cdot\int\limits_0^\theta I_j(\theta')P_{\nu_j}[\cos(\pi-\theta')]\sin\theta'\,d\theta'.$$

It should satisfy conditions of limitedness of function $\Phi_j(\xi)$ at points $\theta = 0$ and $\theta = \pi$. Fulfillment of this condition at $\theta = 0$ occurs only if $C_j = 0$. Indeed, when $\theta = 0$

$$\Phi_j(\xi)=C_jP_{\nu_j}(\cos\pi)+D_jP_{\nu_j}(\cos 0).$$

Since function $P_{\nu j}[\cos(\pi - \theta)]$ with $\theta \to 0$, has logarithmic peculiarity, it is necessary that $C_j = 0$.

At point $\theta = \pi$ general solution will have form

$$\Phi_j(\xi)=P_\nu(\cos\pi)\left\{D_j+\frac{2\pi^2}{cN_j\sin\nu_j\pi}\int\limits_0^\pi I_j(\theta')P_{\nu_j}[\cos(\pi-\theta')]\sin\theta'\,d\theta'\right\}-$$

$$-\frac{2\pi^2}{cN_j\sin\nu_j\pi}P_{\nu_j}(\cos 0)\int\limits_0^\pi I_j(\theta')P_{\nu_j}(\cos\theta')\sin\theta'\,d\theta'.$$

It will be limited at this point if coefficient for $P_\nu(\cos\pi)$ is equal to 0, whence

$$D_j=-\frac{2\pi^2}{cN_j\sin\nu_j\pi}\int\limits_0^\pi I_j(\theta')P_{\nu_j}[\cos(\pi-\theta')]\sin\theta'\,d\theta'.$$

Thus solution of equation (2.81) will be

$$\Phi_j(\theta)=-\frac{2\pi^2}{cN_j\sin\nu_j\pi}\left\{P_{\nu_j}[\cos(\pi-\theta)]\int\limits_0^\theta I_jP_{\nu_j}[\cos\theta']\sin\theta'\,d\theta'+\right.$$

(2.89)

$$+P_{\nu_j}[\cos\theta]\int_{\theta}^{\pi} I_j\cdot P_{\nu_j}[\cos(\pi-\theta')]\sin\theta' d\theta'\Big\}. \qquad (2.89 \text{ cont'd})$$

Complete solution of initial equation (2.76), according to (2.77), will have form

$$\begin{vmatrix}B\\A\end{vmatrix} = -\frac{2\pi^2}{c}\sum\begin{vmatrix}Y_j(r)\\Z_j(r)\end{vmatrix}\frac{1}{\sin\nu_j\pi\cdot N_j}\Big\{P_{\nu_j}[\cos(\pi-\theta)]\times$$

$$\times\int_0^{\theta} I_j\cdot P_{\nu_j}[\cos\theta']\sin\theta' d\theta' + P_{\nu_j}[\cos\theta]\cdot\int_{\theta}^{\pi} I_j P_{\nu_j}[\cos(\pi-\theta')]\sin\theta' d\theta'\Big\}. \qquad (2.90)$$

In particular case, when field $\begin{vmatrix}B\\A\end{vmatrix}$ is excited by vertical Hertz doublet, located on polar axis $\theta = 0$ at point $r = b$,

$$\begin{vmatrix}B\\A\end{vmatrix} = -\frac{\pi P}{cb^3}\sum_{j=0}^{\infty}\begin{vmatrix}Y_j(r)\\Z_j(r)\end{vmatrix}\frac{Y_j(b)}{N_j\sin\nu_j\pi}\cdot P_{\nu_j}[\cos(\pi-\theta)], \qquad (2.91)$$

where P is electrical moment of dipole.

Phase function $\Phi_j(\theta)$ in this case has the form

$$\Phi_j = -\frac{\pi P Y_j(b)}{b^3 c N_j \sin\nu_j\pi}\cdot P_{\nu_j}[\cos(\pi-\theta)]. \qquad (2.89')$$

In order to consider field at large distances θ from Hertz doublet, we use formulas (2.71) and (2.72) and present $P_{\nu_j}[\cos(\pi-\theta)]$ in the form

$$P_{\nu_j}[\cos(\pi-\theta)] = \frac{1}{\pi i}\Big\{L^{(1)}_{\nu_j}[\cos(\pi-\theta)] - L^{(2)}_{\nu_j}[\cos(\pi-\theta)]\Big\}.$$

Further we expand $\frac{1}{\sin\nu_j\pi}$ in series after formula of geometric progression

$$\frac{1}{\sin\nu_j\pi} = -2ie^{\nu_j\pi i}\Big[1+e^{2\pi\nu_j i}+\dots e^{2\pi n\nu_j i}+\dots\Big].$$

This series converges, since $\left|e^{2\pi\nu_j i}\right| < 1$, if we take principal values of roots ν_j. Thus we present (2.91) in the form

$$\begin{vmatrix}B\\A\end{vmatrix} = \frac{2P}{cb^3}\sum_{j=0}^{\infty}Y_j(b)\begin{vmatrix}Y_j(r)\\Z_j(r)\end{vmatrix}e^{\nu_j\pi i}\frac{L^{(1)}_{\nu_j}-L^{(2)}_{\nu_j}}{N_j}\sum_{n=0}^{\infty}e^{2\pi n\nu_j i}.$$

Switching to asymptotic representation of functions $L^{(1)}_{\nu j}$ and $L^{(2)}_{\nu j}$, which is justified for sufficiently large θ, we obtain

$$e^{-i\omega t}\begin{vmatrix}B\\A\end{vmatrix} = \frac{2P}{cb^3}\sqrt{\frac{\pi}{2\sin\theta}}\sum_{j=0}^{\infty}Y_j(b)\begin{vmatrix}Y_j(r)\\Z_j(r)\end{vmatrix}\frac{\nu_j^{-\frac{1}{2}}}{N_j}e^{-i\omega t}\times$$

$$\times\left\{\sum_{n=0}^{\infty}e^{i\left[(n+1)2\pi\nu_j-\left(\nu_j\theta+\frac{\pi}{4}\right)\right]}+e^{i\left[\nu_j\theta+\frac{\pi}{4}+2\pi n\nu_j\right]}\right\}. \quad (2.92)$$

From (2.92) one may see that second members in brackets represent normal waves arriving at point of reception (r, θ) from radiator $\theta = 0$. Where wave $n = 0$ passes over the shortest arc of great circle, lagging in phase by $R_e(\nu_j{}^*)\ \theta$, and waves c $n \neq 0$ pass n times around globe reaching point (r, θ); they are called direct echoes.

First members in brackets are normal waves coming to point of reception $(r_1\ \theta)$ over larger arc of great circle $(n = 0)$, having first circled the globe n times (inverse echoes).

If ν_j have sufficiently large imaginary parts, direct and inverse echoes, and also first reverse wave, are considerably weaker than normal waves coming to point of reception by the shortest means, and it is possible to disregard them. Then approximate solution (2.92) we record in the form

$$\begin{vmatrix}B\\A\end{vmatrix}e^{-i\omega t} \approx \frac{2P}{cb^3}\sqrt{\frac{\pi}{2\sin\theta}}e^{-i\omega t}\sum_{j=0}^{\infty}Y_j(b)\begin{vmatrix}Y_j(r)\\Z_j(r)\end{vmatrix}\frac{e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)}}{N_j\sqrt{\nu_j}}. \quad (2.93)$$

§ 6. Calculation Formulas for Components of Electromagnetic Field in Layer of Atmosphere and Under Surface of Earth (Model B)

We shall use (2.93) subsequently for calculation of electromagnetic field in layer of atmosphere (r_0, r_1) and under water or earth's surface $(r < r_0)$. In both these layers media are isotropic; therefore components of electromagnetic field are calculated by simplified formulas (2.25)-(2.30), with $\varepsilon_{\theta\varphi} = -\varepsilon_{\varphi\theta} = 0$.

For every normal wave of number j

$$H_{\varphi j} = \frac{1}{r}\frac{\partial B_j}{\partial\theta} = \frac{Y_j}{r}\frac{d\Phi_j}{d\theta}; \qquad E_{\varphi j} = \frac{1}{r}\frac{\partial A_j}{\partial\theta} = \frac{Z_j}{r}\frac{d\Phi_j}{d\theta}$$

$$E_{rj} = -\frac{i\nu_j^2}{k_1r^3}B_j = -\frac{i\nu_j^2Y_j\Phi_j}{k_1r^3}; \qquad H_{rj} = \frac{i\nu_j^2}{k_1r^3}A_j = \frac{i\nu_j^2Z_j\Phi_j}{k_1r^3} \quad (2.94)$$

$$E_{\theta_j} = -\frac{i}{k_1 r}\frac{\partial^2 B_j}{\partial r \partial \theta} = -\frac{i}{k_1 r}\frac{dY_j}{dr}\cdot\frac{d\Phi_j}{d\theta};\; H_{\theta_j} = \frac{i}{k_1 r}\frac{\partial^2 A_j}{\partial r \partial \theta} = \frac{i}{k_1 r}\frac{dZ_j}{dr}\cdot\frac{d\Phi_j}{d\theta}. \quad \text{(2.94 cont'd)}$$

Here is used equation (2.52).

As can be seen, components of field are divided into two groups; H_φ, E_r, E_θ are determined through scalar function of **B**, and E_φ, H_r, H_θ are determined through scalar function of **A**.

Using (2.89') and asymptotic representation $P_\nu[\cos(\pi - \theta)]$, we obtain distant field in components (in CGS):

$$\left.\begin{aligned}
H_\varphi &= \frac{2Pi}{c\vartheta^2 r}\sqrt{\frac{\pi}{2\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty}\frac{Y_j(b)Y_j(r)}{N_j}\nu_j^{\frac{1}{2}} e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
E_r &= -\frac{2Pi}{c\vartheta^2 k_1 r^2}\sqrt{\frac{\pi}{2\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty}\frac{Y_j(b)Y_j(r)}{N_j}\nu_j^{\frac{3}{2}} e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
E_\theta &= \frac{2P}{c\vartheta^2 k_1 r}\sqrt{\frac{\pi}{2\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty}\frac{Y_j(b)Y_j'(r)}{N_j}\nu_j^{\frac{1}{2}} e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},
\end{aligned}\right\} \quad (2.95)$$

$$\left.\begin{aligned}
E_\varphi &= \frac{2Pi}{c\vartheta^2 r}\sqrt{\frac{\pi}{2\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty}\frac{Y_j(b)Z_j(r)}{N_j}\nu_j^{\frac{1}{2}} e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
H_r &= \frac{2Pi}{c\vartheta^2 k_1 r^2}\sqrt{\frac{\pi}{2\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty}\frac{Y_j(b)Z_j(r)}{N_j}\nu_j^{\frac{3}{2}} e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
H_\theta &= -\frac{2P}{c\vartheta^2 k_1 r}\sqrt{\frac{\pi}{2\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty}\frac{Y_j(b)Z_j'(r)}{N_j}\nu_j^{\frac{1}{2}} e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)}.
\end{aligned}\right\} \quad (2.96)$$

Here $\nu_j = \nu_j^*;\; Y_j' = \dfrac{dY_j}{dr};\; Z_j' = \dfrac{dZ_j}{dr}.$

For calculation of field in layer of atmosphere and in earth it is necessary to make further concretization of formulas (2.95) and (2.96). Let us introduce following analytic functions of two arguments:

$$D_\nu(x,y) = \begin{vmatrix} J_\nu(kx) & n_\nu(kx) \\ J_\nu(ky) & n_\nu(ky) \end{vmatrix},$$

$$D_\nu(x',y) = \begin{vmatrix} J'_\nu(kx) & n'_\nu(kx) \\ J_\nu(ky) & n_\nu(ky) \end{vmatrix},$$

$$D_\nu(x,y') = \begin{vmatrix} J_\nu(kx) & n_\nu(kx) \\ J'_\nu(ky) & n'_\nu(ky) \end{vmatrix}, \quad (2.97)$$

$$D_\nu(x',y') = \begin{vmatrix} j'_\nu(kx) & n'_\nu(kx) \\ j'_\nu(ky) & n'_\nu(ky) \end{vmatrix}, \quad (2.97 \text{ cont'd})$$

where j_ν and n_ν are modified Bessel and Neumann functions of order ν, while j'_ν and n'_ν are their derivatives with respect to argument. Then in layer of atmosphere (r_0, r_1):

$$\begin{aligned} Y_j(r) &= [D_{\nu_j}(a',r) - \tilde{Z}_y D_{\nu_j}(a,r)], \\ Z_j(r) &= \varkappa_j\left[D_{\nu_j}(a,r) - \frac{1}{\tilde{Z}_z} \cdot D_{\nu_j}(a',r)\right], \end{aligned} \quad (2.98)$$

where

$$\tilde{Z}_y = \frac{k_1}{k_0}\frac{j_\nu'(k_0 a)}{j_\nu(k_0 a)} \text{ and } \tilde{Z}_z = \frac{k_0}{k_1}\frac{j_\nu'(k_0 a)}{j_\nu(k_0 a)} \quad (2.99)$$

are impedances of earth to spherical waves of vertical ($\tilde{Z}_y$) and horizontal ($\tilde{Z}_z$) polarizations. When $\sigma_0 \to \infty$, wave number of layer of earth $k_0 \to \infty$ and $\tilde{Z}_y \to 0$, while $\tilde{Z}_z \to \infty$.

Coefficient of polarization $\varkappa_j(r)$ of normal wave of number j determines ratio of intensities of components (2.96) to components (2.95). It is calculated during the finding of eigenvalues and functions of operator of normal waves.

Taking into account (2.98), designating

$$\begin{aligned} \overline{D_\nu(a,r)} &= D_\nu(a,r) - \frac{1}{\tilde{Z}_z} D_\nu(a',r), \\ \overline{D_\nu(a',r)} &= D_\nu(a',r) - \tilde{Z}_y D_\nu(a,r), \\ \overline{D_\nu(a,r')} &= D_\nu(a,r') - \frac{1}{\tilde{Z}_z} D_\nu(a',r'), \\ \overline{D_\nu(a',r')} &= D_\nu(a',r') - \tilde{Z}_y D_\nu(a,r') \end{aligned} \quad (2.100)$$

and switching to system of MKS units, we record component of electromagnetic field in layer of atmosphere in the following form:

$$\left.\begin{aligned} H_\varphi &\approx iA_0'\sqrt{\frac{W}{\sin\theta}}\, e^{-i\omega t}\sum_{j=0}^{\infty}\varkappa_j\, \overline{D_{\nu_j}(a',b)} \cdot \overline{D_{\nu_j}(a',r)}\, e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)} \cdot \frac{k_1 r}{\nu_j}, \\ E_r &\approx -iA_0\sqrt{\frac{W}{\sin\theta}}\, e^{-i\omega t}\sum_{j=0}^{\infty}\varkappa_j\, \overline{D_{\nu_j}(a'b)}\, \overline{D_{\nu_j}(a',r)}\, e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)}, \\ E_\theta &\approx A_0\sqrt{\frac{W}{\sin\theta}}\, e^{-i\omega t}\sum_{j=0}^{\infty}\varkappa_j\, \overline{D_{\nu_j}(a',b)} \cdot \overline{D_{\nu_j}(a',r')}\, e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)} \cdot \frac{k_1 r}{\nu_j}, \end{aligned}\right\} \quad (2.101)$$

$$
\left.
\begin{aligned}
E_\varphi &\simeq iA_0\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} a_j\varkappa_j\,\overline{D_{\nu_j}(a',b)}\;\overline{D_{\nu_j}(a,r)}\,e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)}\frac{k_1 r}{\nu_j},\\
H_r &\simeq iA_0'\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} a_j\varkappa_j\,\overline{D_{\nu_j}(a',b)}\;\overline{D_{\nu_j}(a,r)}\,e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
H_\theta &\simeq -A_0'\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} a_j\varkappa_j\,\overline{D_{\nu_j}(a',b)}\;\overline{D_{\nu_j}(a,r')}\,e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)}\frac{k_1 r}{\nu_j},
\end{aligned}
\right\}
\qquad (2.102)
$$

where E is in μv/m and H is in μa/m,

W – emissive power, in kilowatts,

λ – wavelength in meters,

$A_0 = 0{,}1829 \cdot 10^{-20}$; $A'_0 = \dfrac{A_0}{120\pi}$, (2.103)

$a_j = \lambda\sqrt{\dfrac{\pi}{N_j}}\,\nu_j$ – coefficients of excitation of wave of number j,

$\overline{D_{\nu_j}(a',b)}$ – height factor at point of radiation

$\overline{D_{\nu_j}(a,r)}$, $\overline{D_{\nu_j}(a',r)}$, $\overline{D_{\nu_j}(a,r')}$ and $\overline{D_{\nu_j}(a',r')}$ – height factors at point of recpetion,

$\varkappa_j$ – polarization factor.

Formulas (2.101)-(2.102) for surface of earth (r = a) with Hertz doublet also located on surface of earth (b = a) take form:

$$
\left.
\begin{aligned}
H_\varphi(a,\theta) &\simeq iA_0'\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} a_j e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
E_r(a,\theta) &\simeq -iA_0\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} a_j e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
E_\theta(a,\theta) &\simeq iA_0\frac{k_1}{k_0}\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} a_j e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},
\end{aligned}
\right\}
\qquad (2.104)
$$

$$
\left.
\begin{aligned}
E_\varphi(a,\theta) &\simeq A_0\frac{k_1}{k_0}\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} a_j\varkappa_j e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
H_r(a,\theta) &\simeq A_0'\frac{k_1}{k_0}\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} a_j\varkappa_j e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
H_\theta(a,\theta) &\simeq -A_0'\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} a_j\varkappa_j e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)}
\end{aligned}
\right\}
\qquad (2.105)
$$

For calculation of electromagnetic field under surface of earth or ocean we determine Y_j and Z_j in interval (0, a):

$$Y_j(r) = C\frac{J_{\nu_j}(k_0 r)}{J_{\nu_j}(k_0 a)},$$

$$Z_j(r) = C\varkappa_j \frac{1}{Z_0}\frac{J_{\nu_j}(k_0 r)}{J_{\nu_j}(k_0 a)}.$$

For approximation we consider

$$\frac{J_\nu(k_0 r)}{J_\nu(k_0 a)} \simeq e^{ik_0(a-r)} = e^{ik_0 H},$$

where

$H = a - r$ is distance from point of reception to surface of ocean or earth

$$k_0 = k_1\sqrt{\varepsilon_0' + \frac{4\pi\sigma_0 i}{\omega}} \simeq \delta(1+i),$$

$$\delta = \frac{1}{c}\sqrt{2\pi\sigma_0\omega}, \tag{2.106}$$

δ is attenuation factor of flat electromagnetic waves of frequency ω penetrating medium with conductivity σ_0.

Then electromagnetic field at depth H is equal to:

$$\left.\begin{aligned}
H_\varphi(H,\theta) &\simeq iA_0'\sqrt{\frac{W}{\sin\theta}}\, e^{-i\omega t} e^{-\delta(1-i)H}\sum_{j=0}^{\infty} n_j^2 \overline{D_{\nu_j}(a',b)}\, e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
E_r(H,\theta) &\simeq iA_0\sqrt{\frac{W}{\sin\theta}}\, e^{-i\omega t}\left(\frac{k_1}{k_0}\right)^2 e^{-\delta(1-i)H}\sum_{j=0}^{\infty} n_j^2 \overline{D_{\nu_j}(a',b)}\, e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
E_\theta(H,\theta) &\simeq iA_0\sqrt{\frac{W}{\sin\theta}}\, e^{-i\omega t}\frac{k_1}{k_0} e^{-\delta(1-i)H}\sum_{j=0}^{\infty} n_j^2 \overline{D_{\nu_j}(a',b)}\, e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},
\end{aligned}\right\} \tag{2.107}$$

$$\left.\begin{aligned}
E_\varphi &\simeq A_0\sqrt{\frac{W}{\sin\theta}}\, e^{-i\omega t}\frac{k_1}{k_0} e^{-\delta(1-i)H}\sum_{j=0}^{\infty} n_j^2 \varkappa_j \overline{D_{\nu_j}(a',b)}\, e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
H_r &\simeq A_0'\sqrt{\frac{W}{\sin\theta}}\, e^{-i\omega t}\frac{k_1}{k_0} e^{-\delta(1-i)H}\sum_{j=0}^{\infty} n_j^2 \varkappa_j \overline{D_{\nu_j}(a',b)}\, e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)},\\
H_\theta &\simeq -A_0'\sqrt{\frac{W}{\sin\theta}}\, e^{-i\omega t} e^{-\delta(1-i)H}\sum_{j=0}^{\infty} n_j^2 \varkappa_j \overline{D_{\nu_j}(a',b)}\, e^{i\left(\nu_j\theta+\frac{\pi}{4}\right)}.
\end{aligned}\right\} \tag{2.108}$$

From expressions (2.106)(2.108) it follows that because of great ground conductivity, all components of field rapidly diminish with submersion of point of reception in water per the same exponential law with attenuation factor δ.

§ 7. Generalization of Spectral Theory of VLW Propagation in Case of Dependence of Parameters of Medium on Geographic Coordinates (Model C) (Method of Modulated Normal Waves)

Solution of problem of VLW propagation on the basis of model C is intimately connected with solution of problem on the basis of model B, considered in §§ 2-6 of this chapter.

According to § 1, we consider that model C is characterized by layers with radii r_k; k = 0, 1, 2, 3, ... N - 1 weakly depend on geographic coordinates θ and φ. Let us record this fact in the following way:

$$r_k = r_k(\mu\theta, \mu\varphi), \tag{2.109}$$

where μ is small parameter. Coordinates θ and φ enter in function r_k only in combinations $\mu\theta$ and $\mu\varphi$, which testifies to small changes of r_k along coordinates θ and φ during the period of wave.

We also consider that ε^k_{rr}, $\varepsilon^k_{\theta\theta}$ and $\varepsilon^k_{\theta\varphi}$ depend on $\mu\theta$ and $\mu\varphi$, that is, slowly change from coordinates θ and φ. Under these conditions, for model C it is possible to introduce vector function $\begin{vmatrix} B \\ A \end{vmatrix}$, connected with components of field according to (2.25)-(2.30) and determined approximately by operational equation

$$L_r \begin{vmatrix} B \\ A \end{vmatrix} + L_{\theta\varphi} \begin{vmatrix} B \\ A \end{vmatrix} = \frac{4\pi}{c} r^2 \begin{vmatrix} I_r \\ O \end{vmatrix}, \tag{2.110}$$

where L_r is operator generated by differential-matrix expressions (2.40) and also by conditions of jumps (2.42)-(2.43) and boundary conditions (2.44)-(2.45). Operator $L_{\theta\varphi}$ is generated by differential expression

$$l_{\theta\varphi} = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\cdot\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}. \tag{2.111}$$

and conditions of limitedness of solutions for $\theta = 0$ and $\theta = \pi$.

We look for solution of equation without right side in the form

$$\begin{vmatrix} B \\ A \end{vmatrix} = \begin{vmatrix} Y(r;\ \mu\theta,\ \mu\varphi) \\ Z(r;\ \mu\theta,\ \mu\varphi) \end{vmatrix} \cdot \Psi(\theta, \varphi). \tag{2.112}$$

Putting (2.112) in (2.110), for $I_r = 0$, and dividing variables, we obtain

$$\frac{L_r\begin{vmatrix}Y\\Z\end{vmatrix}}{\begin{vmatrix}Y\\Z\end{vmatrix}} = -\frac{L_{\theta\varphi}\cdot\Psi}{\Psi} = \chi. \tag{2.113}$$

When $\mu = 0$ (model B) χ was constant, since exact splitting of equation (2.113) in two took place. Now with small μ it is possible to consider that constant χ is slowly changing function of coordinates θ and φ

$$\chi = \chi(\mu\theta, \mu\varphi). \tag{2.114}$$

Then we obtain from (2.113) equation in eigenvalues of χ of operator of normal waves in the form

$$L_r\begin{vmatrix}Y\\Z\end{vmatrix} = \chi(\mu\theta, \mu\varphi)\begin{vmatrix}Y\\Z\end{vmatrix}. \tag{2.115}$$

Eigenvalues of χ_j

$$\chi_0, \chi_1, \chi_2 \cdots \chi_j \cdots \tag{2.116}$$

weakly depend on coordinates θ and φ. They have to be calculated for every point (θ, φ) by means of solution of problem (2.53)-(2.57), in which parameters ε^k_{rr}, $\varepsilon^k_{\theta\theta}$, $\varepsilon^k_{\theta\varphi}$ and r_k are determined for point (θ, φ) with respect to assigned functions

$$\varepsilon^k_{rr}(\mu\theta, \mu\varphi);\ \varepsilon^k_{\theta\theta}(\mu\theta, \mu\varphi);\ \varepsilon^k_{\theta\varphi}(\mu\theta, \mu\varphi);\ r_k(\mu\theta, \mu\varphi).$$

To every eigenvalue of χ_j corresponds eigenfunction

$$\begin{vmatrix}Y_j(r;\mu\theta, \mu\varphi)\\Z_j(r;\mu\theta, \mu\varphi)\end{vmatrix} \quad j = 0, 1, 2, 3 \dots \infty,$$

slowly changing, depending upon coordinate θ and φ.

From (2.113) we obtain also equation

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi_j}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\Psi_j}{\partial\varphi^2} + \chi_j(\mu\theta, \mu\varphi)\Psi_j = 0, \tag{2.117}$$

determining function of phase $\Psi_j(\theta, \varphi)$

Free normal waves (2.112) have form

$$\begin{vmatrix}B_j\\A_j\end{vmatrix} = \begin{Vmatrix}Y_j(r;\mu\theta, \mu\varphi)\\Z_j(r;\mu\theta, \mu\varphi)\end{Vmatrix}\Psi_j(\theta,\varphi). \tag{2.118}$$

They were called [36, 37] modulated normal waves. We borrowed this term from work of S. M. Rytov [42], since operator of modulated normal waves is analogous to operator of modulated normal oscillations.

Applying method of geometric optics, we seek solution for phase factor in the form

$$\Psi_j = U_j(\theta, \varphi) e^{iS_j(\theta, \varphi)}; \qquad j = 0, 1, 2, 3 \ldots \infty, \tag{2.119}$$

where $U_j(\theta, \varphi)$ are slowly changing functions of θ and φ.

Then for functions S_j and U_j we obtain following equations:

$$\left(\frac{\partial S_j}{\partial \theta}\right)^2 + \frac{1}{\sin^2\theta}\left(\frac{\partial S_j}{\partial \varphi}\right)^2 = \chi_j(r_0, \theta, \varphi), \quad j = 0, 1, 2, 3 \ldots \infty, \tag{2.120}$$

which are Eikonal equations, and

$$\frac{\partial}{\partial \theta}\left(U_j^2 \sin\theta \frac{\partial S_j}{\partial \theta}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \varphi}\left(U_j^2 \frac{\partial S_j}{\partial \varphi}\right) = 0 \quad j = 0, 1, 2, 3 \ldots \infty, \tag{2.121}$$

expressing law of conservation of energy.

Thus in approximation of geometric optics modulated normal waves have form

$$\begin{vmatrix} B_j \\ A_j \end{vmatrix} e^{-i\omega t} = \begin{vmatrix} Y_j(r, r_0, \theta, \varphi) \\ Z_j(r, r_0, \theta, \varphi) \end{vmatrix} U_j(\theta, \varphi) e^{iS_j(\theta, \varphi)} e^{-i\omega t}. \tag{2.122}$$

Equation (2.120) for phase S_j of normal wave determines propagation of "radio beam" in two-dimensional spherical medium, determined by coordinates (θ, φ) and characterized by local "angular refractive index" $\sqrt{-\chi_j}$, which changes slowly over spherical surface. For every normal wave of number j there is particular refractive index $\sqrt{-\chi_j}$. Therefore "radio beams" belonging to different normal waves pass on different trajectories. In Fig. 3 are conditionally shown trajectories of "radio beams" for j = 0, 1, 2, ..., connecting point of radiation $O(\theta = 0)$ with point of reception P (θ, φ). Dotted line shows geodesic line.

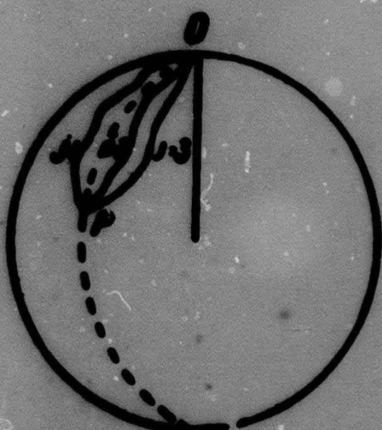


Fig. 3.

Quantity S_j is expressed by integral

$$S_j = \int_0^P \sqrt{-\chi_j(\theta, \varphi)}\, ds, \tag{2.123}$$

taken on radio beam connecting point of radiation O with point of reception P. S_j determines phase of normal wave of number j, traveling on this path. Here ds is angular element of arc, taken along beam.

Distortion of "beam" is determined by usual law of refraction, which follows from (2.120). Calculation shows that for first types of normal waves, under the condition that radio route does not pass along twilight line, distortion of beams is so insignificant that it may be disregarded. Therefore, considering that "beam" spreads along geodesic line, we determine phase S_j ($j = 0, 1, 2$) by integral

$$S_j \simeq \int_0^P \sqrt{-\chi_j(\theta, \varphi)}\, d\theta. \tag{2.124}$$

Putting (2.124) in (2.121), we obtain amplitude factor U_j in the form:

$$U_j = \frac{C_j}{\sqrt{\nu_j \sin\theta}}, \tag{2.125}$$

$$\nu_j \simeq \sqrt{-\chi_j}.$$

Then we record solution of (2.122) in the form of modulated normal wave

$$\begin{vmatrix} B_j \\ A_j \end{vmatrix} e^{-i\omega t} \simeq \begin{vmatrix} Y_j \\ Z_j \end{vmatrix} \frac{C_j \nu_j^{-\frac{1}{2}}}{\sqrt{\sin\theta}} e^{i\left(\omega t \pm \int_0^\theta \nu_j d\theta\right)}. \tag{2.126}$$

When $\mu = 0$, this solution becomes asymptotic representation of normal waves of model B already obtained in § 5.

Solution (2.126) is applicable both to "frozen" and also to slowly time-varying ionosphere. In last case χ_j may be represented in the form of sum

$$\chi_j = \overline{\chi}_j + \delta\chi_j,$$

where $\overline{\chi}_j$ is time-averaged value of χ_j, and $\delta\chi_j$ is variable part.

Accordingly, in formula (2.126) it is necessary to introduce

$$\nu_j = \overline{\nu}_j + \delta\nu_j.$$

Variation of parameters of ionosphere can lead to distortions of beams, slowly varying with time. We shall disregard this effect, since it depends on member $\partial^2\nu_j/\partial\varphi^2$, which, as it is easy to show, is minute.

It is necessary to note that in case of slowly changing ionosphere equations (2.120) and (2.121) are applicable only under condition that space modulation of ionosphere has shorter periods than does temporal modulation, that is

$$\chi_j = \chi_j(\mu\theta, \mu\varphi, \mu^2 t),$$

where μ is small parameter.

In our case periods of space modulation for wavelength $\lambda_0 \cong 20$ km is 10-15 times greater than λ_0, while periods of temporal modulation are greater than period $T = \frac{2\pi}{\omega}$ by at least 1000 times.

*Linear refractive index is equal to $\frac{1}{a}\nu_j \simeq \frac{1}{a}\sqrt{-\chi_j}$.

Now let us return to finding solution of nonuniform equation (2.110). Let us represent it in the form of sum of modulated normal waves

$$\begin{vmatrix} B \\ A \end{vmatrix} e^{-i\omega t} = \sum_{j=0}^{\infty} \begin{vmatrix} Y_j(r, \mu\theta, \mu\varphi) \\ Z_j(r, \mu\theta, \mu\varphi) \end{vmatrix} \Phi_j(\theta, \varphi) e^{-i\omega t}.$$

Then electromagnetic field excited by vertical Hertz doublet we present in the form

$$\begin{vmatrix} B \\ A \end{vmatrix} e^{-i\omega t} = \frac{2P}{cb^2} \sqrt{\frac{\pi}{2\sin\theta}}\, e^{-i\omega t} \sum_{j=0}^{\infty} Y_j(b) \begin{vmatrix} Y_j(r) \\ Z_j(r) \end{vmatrix} \frac{\nu_j^{-1} \cdot e^{+i\int_0^{\theta} \nu_j d\theta}}{\sqrt{N_j(O)} \cdot \sqrt{N_j(P)}} \tag{2.127}$$

or, using (2.103),

$$\begin{vmatrix} B \\ A \end{vmatrix} e^{-i\omega t} =$$

$$= - A_0 k_1 r^2 e^{-i\omega t} \sqrt{\frac{W}{\sin\theta}} \sum_{j=0}^{\infty} D_{\nu_j}(a'b) \cdot \begin{vmatrix} D_{\nu_j}(a',r) \\ x_j D_{\nu_j}(a,r) \end{vmatrix} \frac{n_j(O) n_j(P)}{\nu_j(O)\nu_j(P)}\, e^{i\int_0^{P} \nu_j d\theta} \tag{2.128}$$

CHAPTER III

INTRODUCTION OF ADDITIONAL INITIAL DATA OBTAINED FROM MEASUREMENTS OF AMPLITUDE AND PHASE OF NEAR FIELD OF VLW AND RESULTS OF THEORY ON IONOSPHERE

§ 1. Data on Near Field

Since parameters of medium $\omega^2_{0,k}$ and $\mathbf{r}_k = \mathbf{r}_0 + h_k$ ($k = 1, 2, 3, \ldots N - 1$) are unknown, then, according to § 1, Chapter I, we solve mixed problem, for which we introduce additional initial data on near field $\mathbf{E}_j$ and $\mathbf{\Psi}_j$ at points $(\mathbf{r}_0, \theta_j)$($j = 0, 1, 2, \ldots N$), located on surface of earth. Such data were obtained in great quantity by English researchers [31, 32, 33, 34, 35], who measured "instantaneous" distribution of amplitudes of near field along earth's surface as function of distance to transmitter [31], and also duirnal variation of amplitude and phase of components of field at fixed points at distances of 90 [32], 200 [33], and 535 km [34] from [VLW] (СДВ) station with call sign GBR (16.0 kilocycles), located near Rugby (52° N, 1° W).* These data are well-known, since investigation of near field has been conducted by Cambridge for more than 25 years and, starting with first works of Hollingworth [52], have been published regularly.

In Figs. 4a and 4b are given typical day and night distributions of amplitude of near field as functions of distance.

In Table 2 are given distances, where there are maximum and minimum field strengths for summer period (day).

*Signals were received on a loop in two positions: in vertical plane passing through points of radiation and reception (normal component), and in vertical plane perpendicular to the above-mentioned (anomalous component of field).

Table 2

	Max	Min	Max	Min	Max	Min
$a\theta$ (km)	112	133	175	224	295	490

Daily variation of amplitude and phase of normal component of field for distance of 90 km is given in Fig. 5a and 5b.

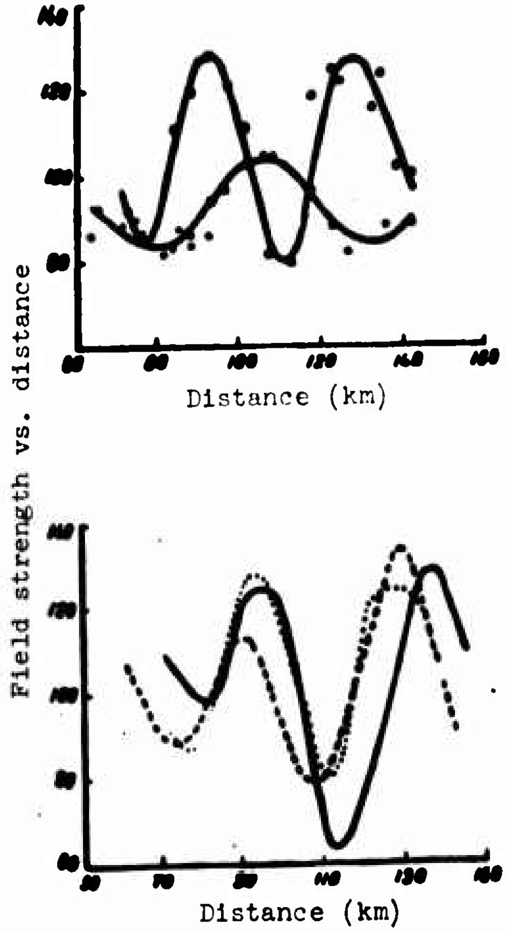
Daily variation of amplitude and phase of normal component of field for distance of 535 km is given in Fig. 6a and 6b.

Analogous dependencies exist for anomalous component (loop perpendicular to plane of propagation) for distances of 90 km. Comparison of normal and anomalous component permits us to separate electromagnetic field reflected from ionosphere and the same way to tie in phase in Figs. 5 and 6 to phase of direct wave [31-35].

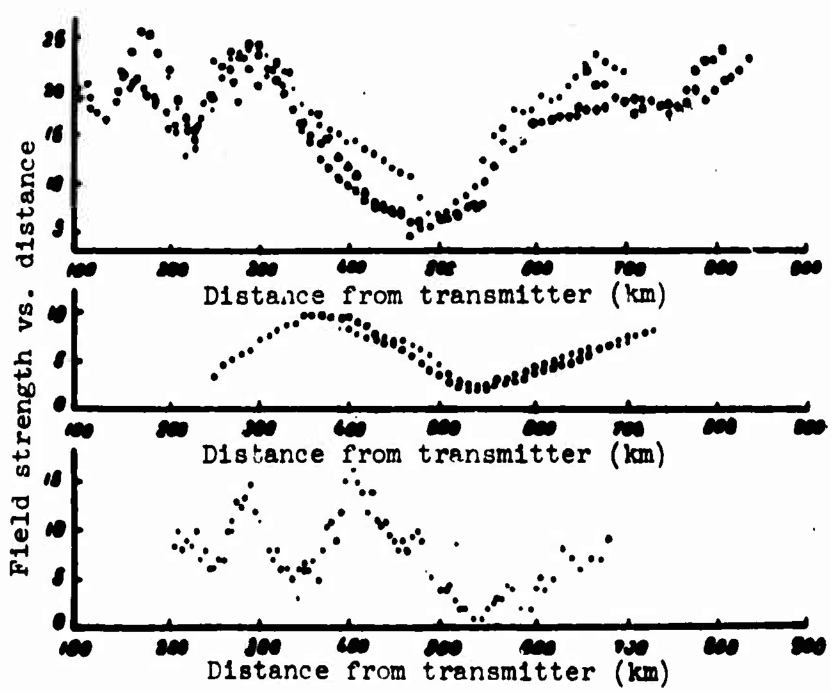
It is necessary to note the exceptional regularity both of space distributions (Fig. 4) so also of daily variation of amplitude and phase of near field (Figs. 5 and 6), which are repeated from day to day, experiencing slow seasonal changes. Exceptions are during periods of flashes of radiation on sun and during magnetic storms, when greater or lesser considerable deviations from above mentioned curves are observed.

§ 2. Method of Solution of Reverse Problem of Theory of VLW Propagation for Near Field

Data on near field depicted in Figs. 5 and 6 pertain to two points with $\theta = 0.014$ radian and $\theta = 0.084$ radian, in which are known amplitudes and phases (E_1, Ψ_1) and (E_2, Ψ_2) at any time of day and year. Thus according to method of solution of mixed problem, presented in § 1, Chapter I, from these data not more than 4 parameters of ionosphere can be determined. Since for every "k" layer of ionosphere we know two parameters $\omega^2_{0,k}$ and $h_k = r_k - r_0$, then data of Figs. 5 and 6 can be used for solution of problem on the basis of two-layered model of ionosphere, for which parameters $\omega^2_{0,1}$, h_1, and $\omega^2_{0,2}$, h_2 are unknown. Data on space distribution of near field (Fig. 4) could be used for solution of problem with thinner structure of ionosphere, but they are incomplete. Therefore we use them only for checking results of solution based on two-layered model and for tying in phase.

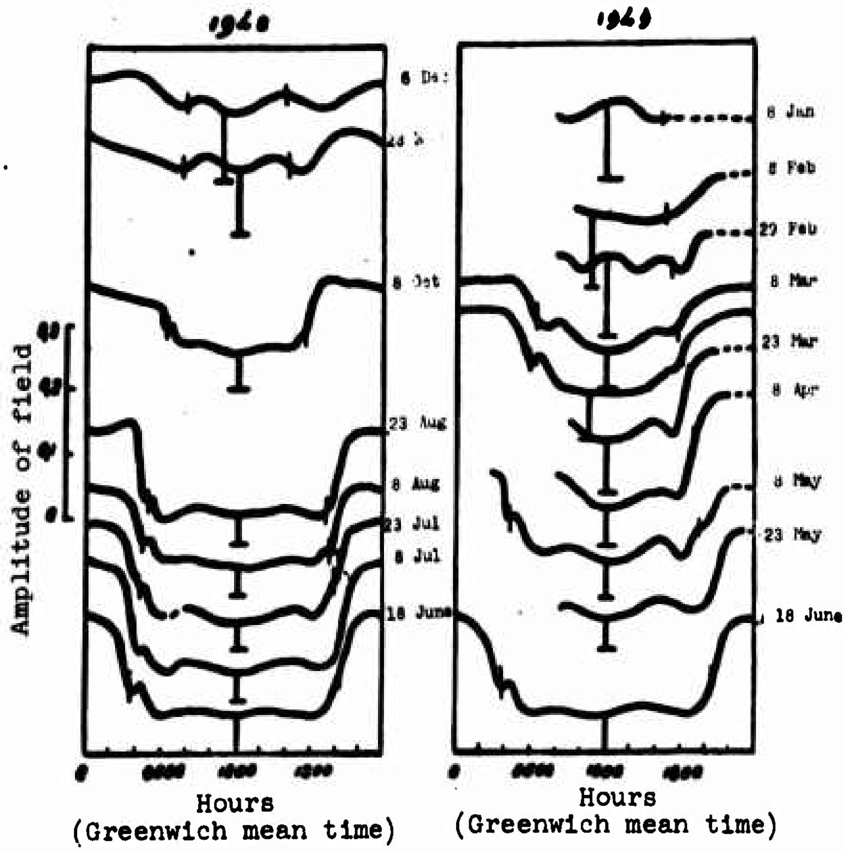

Field strength vs. distance
Distance (km)
Distance (km)

4a


Field strength vs. distance
Distance from transmitter (km)
Distance from transmitter (km)
Distance from transmitter (km)

4b

Fig. 4.


Amplitude of field
8 Dec
8 Oct
23 Aug
8 Aug
23 Jul
8 Jul
18 June
8 Jan
8 Feb
23 Feb
8 Mar
23 Mar
8 Apr
8 May
23 May
18 June
Hours
(Greenwich mean time)
Hours
(Greenwich mean time)

5a

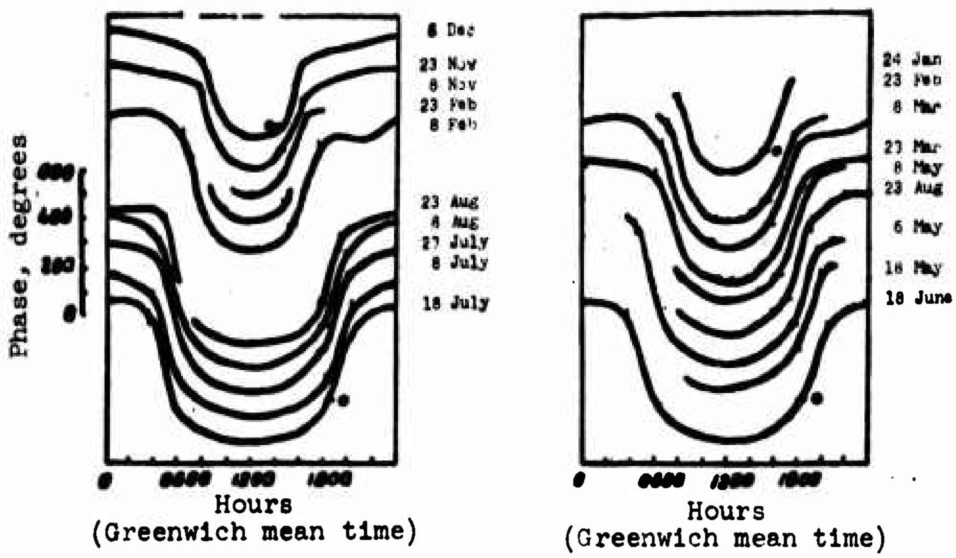
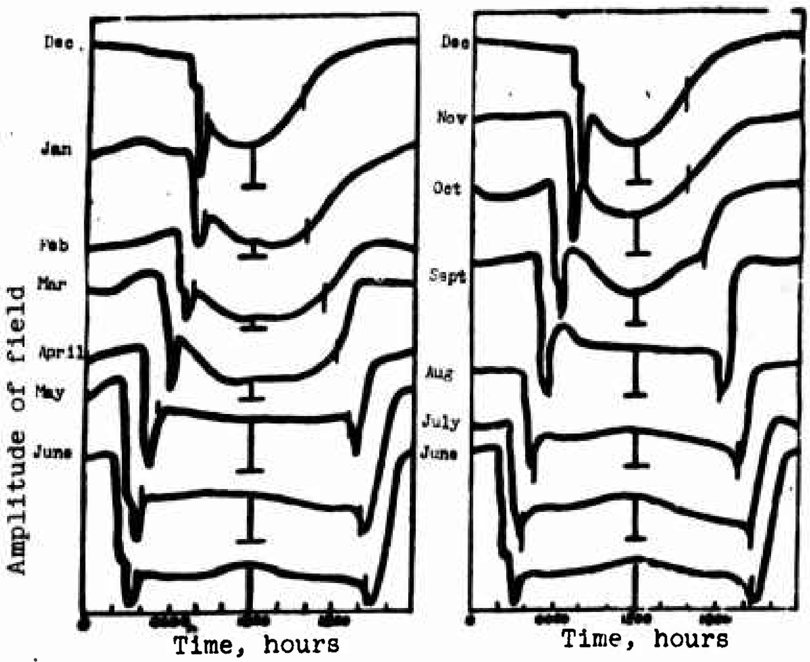
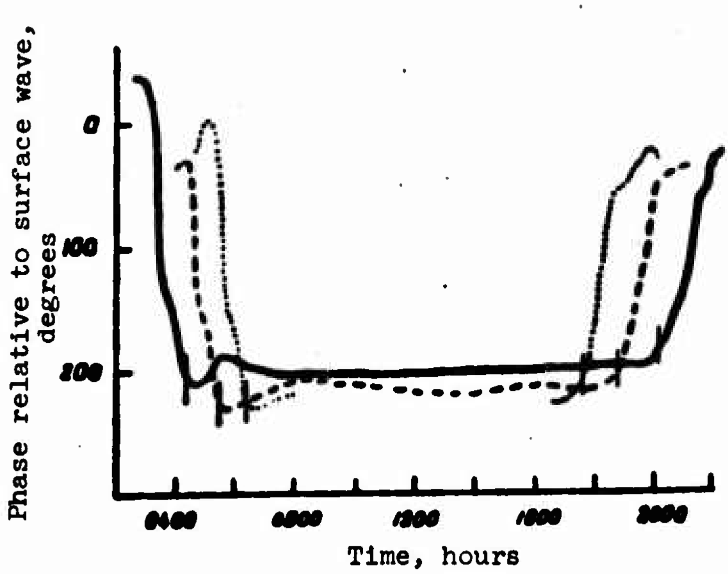

Phase, degrees
8 Dec
23 Nov
8 Nov
23 Feb
8 Feb
23 Aug
8 Aug
27 July
8 July
18 July
24 Jan
23 Feb
8 Mar
23 Mar
8 May
23 Aug
6 May
18 May
18 June
Hours
(Greenwich mean time)
Hours
(Greenwich mean time)

Fig. 5.

5b



6a



6b

Fig. 6.

According to diagram of Fig. 1, we have to solve inverse problem of theory of waves in region of near field, which is reduced to solution of equations (5) and (6) from Chapter I, taking following form:

$$|E_j| = \operatorname{mod} F\left(P, \omega; (a, O); (a, \varphi_j);\ a, \bar{\tau}_e; \omega_H, \nu_{eff}(h_1), \nu_{eff}(h_2): \omega_{0,1}^2, \omega_{0,2}^2, h_1, h_2\right);\quad j = 1, 2, \tag{3.1}$$

$$\Psi_j = \arg F\{P, \omega, (a, 0); (a, \theta_j); a, \bar{\varepsilon}_0; \omega_H, \nu_{eff}(h_1), \nu_{eff}(h_2), \omega^2_{0,1}, \omega^2_{0,2}, h_1, h_2\}; \qquad (3.1\ \text{cont'd})$$
$$j = 1, 2,$$

where P — moment

ω — frequency

(a, 0) — coordinates of Hertz doublet

(a, θ_j) — coordinates of point of reception

a — radius

$\bar{\varepsilon}_0$ — dielectric constant of earth

ω_H, $\nu_{eff}(h_1)$, $\nu_{eff}(h_2)$, $\omega^2_{0,1}$, $\omega^2_{0,2}$, h_1, h_2 — data on first and second layers of ionosphere.

Operation F designates calculation of components of field from data on Hertz doublet and medium. Sought quantities for system of equations (3.1) are $\omega^2_{0,1}$ and h_1, $\omega^2_{0,2}$, and h_2.

Algorithm of calculation of components of field in the form of spectrum of normal waves represented by formulas (2.101)-(2.103), Chapter II, is ineffective for calculation of near field according to data on medium, since number of normal waves which have to be considered for angular distances of θ less than 0.1 radian constitute 1.5-2 tens.

Naturally solution of inverse problem with help of equations (3.1), in which operation F is sum of 15-20 members depending on all four unknowns, presents great difficulties. Therefore we switched from spectral representation (2.101) to beam, in which every component of field, for instance E_r, at point of reception is composed of series of beams experiencing different number of reflections from first and second layer of ionosphere and from earth.

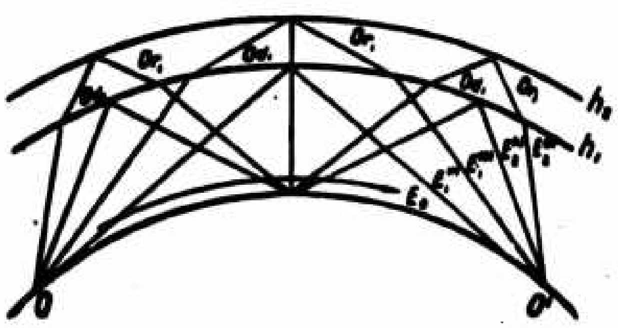


Fig. 7.

Thanks to anisotropy of ionospheric layers, at boundary of first layer and atmosphere $r = r_1 = a + h_1$ there appear two refracted beams, ordinary and extraordinary, designated by letters O_{d_1} and e_{d_1}. On second boundary of ionosphere $r = r_2 = a + h_2$ incident elliptically polarized beam O_{d_1} or e_{d_1} generates two

reflected beams O_{r_1} and e_{r_1}, remaining in first layer, and two refracted beams O_{d_2} and e_{d_2}, passing into second layer (r_2, ∞).

Extraordinary beams experience greater than ordinary; therefore basic role is played by ordinary beams, experiencing small number of reflections and refractions.

In Fig. 7 are depicted several beams remaining ordinary over entire section from radiator to receiving point.

Direct beam for emissive power of 1 kilowatt has intensity

$$E_0 = \frac{300}{D_0} e^{ik_1 D_0} \left(\frac{\mu V}{m}\right); \text{ } D_0 \text{ is length of path of beam.} \qquad (3.2)$$

Beam that has experienced one reflection from lower layer of ionosphere

$$E_1^{(1)} = \frac{150}{D_1} e^{ik_1 D_1} \sin^2\alpha A_1 [1 + r_{BB}]^2 R_{BB}(\beta) \left(\frac{\mu V}{m}\right) \qquad (3.2')$$

where α — angle of incidence on earth

β — angle of incidence on ionosphere

A_1 — coefficient of focusing of ionosphere

D_1 — length of path of beam

r_{BB} — index of reflection from earth of vertically polarized beam

R_{BB} — index of reflection from ionosphere, depending on parameters of ionosphere $\omega^2_{0,1}$, $\nu_{eff}(h_1)$, ω_H and A_1, r_{BB}, R_{BB}, defined in work [5].

Beam experiencing two reflections from lower layer of ionosphere,

$$E_2^{(1)} = \frac{150}{D_2} e^{ik_1 D_2} \sin^2\alpha_2 A_2 [1 + r_{BB}]^2 [R_{BB}^2 r_{BB} + R_{B\Gamma} R_{\Gamma B} r_{\Gamma\Gamma}], \qquad (3.2'')$$

where $R_{B\Gamma}$ and $R_{\Gamma B}$ are conversion factors of polarization during reflection from ionosphere; first index pertains to polarization of incident ray, the second to reflected;

$R_{B\Gamma}$ and $R_{\Gamma B}$ depend on angle of incidence on ionosphere and on properties of first layer;

$r_{\Gamma\Gamma}$ and r_{BB} are indices of reflection of horizontally and vertically polarized beams from earth; remaining designations are analogous to those of formula (3.2).

Beams $E_3^{(1)}$, $E_4^{(1)}$..., and also beams $E_1^{(2)}$, $E_2^{(2)}$..., refracted by first boundary of ionosphere and reflected from second are determined similarly.

Besides ordinary $E_1^{(2)}$, determined by propagation along directions O_{d_1}, O_{r_1} in layer (r_1, r_2), extraordinary beams, propagating in directions e_{d_1}, e_{r_1}, and also mixed beams O_{d_1}, e_{r_1}, and e_{d_1}, O_{r_1} are possible. All of them experience one

reflection from layer r_2 in passing through boundary of layer $r = r_1$. Besides ordinary beam $E_2^{(2)}$, determined by direction o_{d_1}, o_{r_1}, o_{d_1}, o_{r_1}, there are 15 additional beams of mixed type and purely extraordinary beams e_{d_1}, e_{r_1}, e_{d_1}, e_{r_1}, experiencing two reflections from ionosphere.

Full field is composed of infinite number of beams experiencing one, two, three, etc., reflections from ionosphere

$$E = \sum_{k=0}^{\infty} E_k. \tag{3.3}$$

All of them may be calculated from laws of geometric optics, if matrix of indices of reflection from earth is known

$$\|r_3\| = \begin{Vmatrix} r_{oo}, & 0 \\ 0, & r_{rr} \end{Vmatrix} \tag{3.4}$$

and matrix of indices of reflection and refraction of beams on boundaries of layers $r = r_1$ and $r = r_2$.

For lower boundary of ionosphere $r = r_1$ there have to be given indices of reflection

$$\|R^{(1)}\| = \begin{Vmatrix} R^{(1)}_{oo}, & R^{(1)}_{or} \\ R^{(1)}_{ro}, & R^{(1)}_{rr} \end{Vmatrix} \tag{3.5}$$

and refractive indices

$$\|D^{(1)}\| = \begin{Vmatrix} D^{(1)}_{oo}, & D^{(1)}_{or} \\ D^{(1)}_{ro}, & D^{(1)}_{rr} \end{Vmatrix}. \tag{3.6}$$

Analogously, for second boundary of ionosphere $r = r_2$ there have to be known matrices of indices of reflection

$$\|R^{(2)}\| = \begin{Vmatrix} R^{(2)}_{oo}, & R^{(2)}_{or} \\ R^{(2)}_{ro}, & R^{(2)}_{rr} \end{Vmatrix} \tag{3.7}$$

and refraction

$$\|D^{(2)}\| = \begin{Vmatrix} D^{(2)}_{oo}, & D^{(2)}_{or} \\ D^{(2)}_{ro}, & D^{(2)}_{rr} \end{Vmatrix}. \tag{3.8}$$

For calculation of near field of VLW with accuracy to 5% it is sufficient to consider only first five beams depicted in Fig. 7:

$$E \cong E_0 + E_1^{(1)} + E_1^{(2)} + E_2^{(1)} + E_2^{(2)}. \tag{3.9}$$

For small distances (to 100 km) beams E_0 and $E_1^{(2)}$ dominate. For large distances (300-600 km) beams E_0 and $E_1^{(1)}$ dominate.

At distances over 1000 km beam representation (3.3) is impermissible, since for beams with small number of reflections caustic of beams near surface of earth.

Therefore for sliding beams it is necessary to consider phenomenon of diffraction.

As will be seen from Chapters IV and V, the main part of distant field of VLW is created by beams not touching earth's surface. They move on ricochetting trajectories (adhesion effect). Thus distant field of VLW on earth's surface is due only to seepage of waves beneath surface of caustic of beam and therefore carries purely diffractive character.

During calculation of field strength of beams E_0, $E_1^{(1)}$, $E_2^{(1)}$, ... it was assumed that coefficients (3.4)-(3.8) can be approximated by corresponding coefficients for flat medium, but for calculation of length of path of beams D_0, $D_1^{(1)}$, $D_2^{(1)}$, ... sphericity of layers of ionosphere and earth was considered. Disregard for sphericity evokes impermissible error in phase of beams, since with distance of 500 km flat and spherical models give difference in the course of ionospheric beams E of 2-3 km. First model can be applied for calculation of fields only to distance of not more than 300 km.

§ 3. Results of Theory Pertaining to Properties of Ionosphere

Determination of quantities $\omega_{0,1}^2$, $\omega_{0,2}^2$, h_1, and h_2 from equations (3.1) was carried out with help of the presentation of field in the form (3.9) by graphoanalytic method, which will be described in another place. Here we shall give only results of calculation for vernal equinox, which with small changes can be applied to whole spring and summer seasons and partially to autumn.

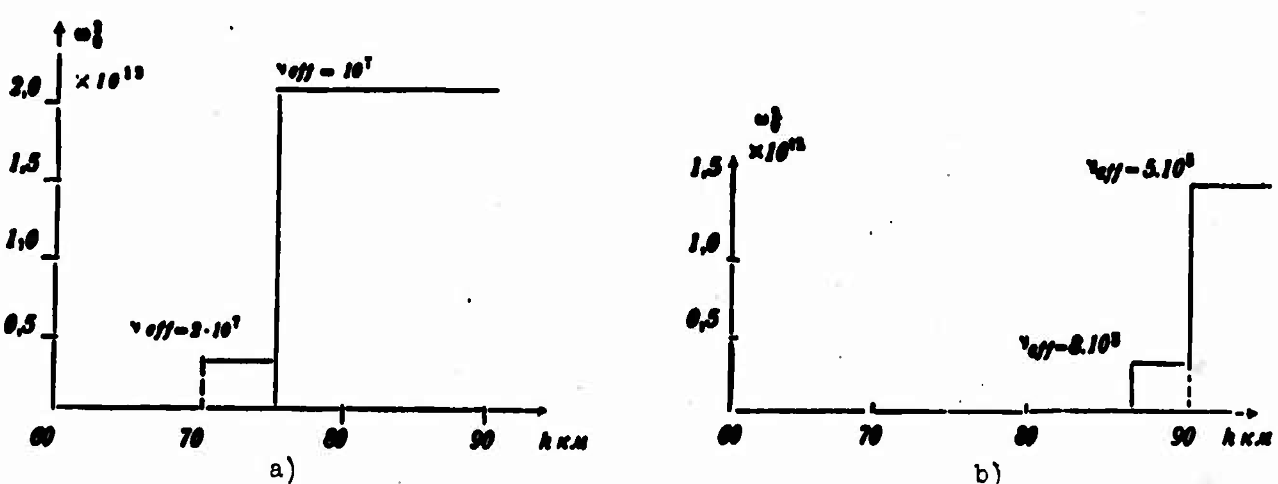


Fig. 8.

On Fig. 8a and 8b are depicted distributions $\omega_{0,1}^2$ and $\omega_{0,2}^2$ obtained from calculation, depending upon height of layer h at noon (8a) and at night (8b).

Corresponding values of ν_{eff} were determined by Nicolet graph, (Chapter II, § 1) and plotted in Fig. 8 on corresponding steps of curve $\omega_0^2(h)$.

Check of two-layered model of ionosphere for daytime distribution of field (Fig. 4) shows what 6 points of maxima and minima of interference curve (3.9), marked in Table II, conform well to diagram depicted in Fig. 8a. This indicates that what actually existing distribution of ionization in daytime is characterized by two regions of height h_1 and h_2 with sharp gradients of ionization. Englishmen [48] call these layers $D_\beta(h = h_1)$ and $D_\alpha(h = h_2)$.

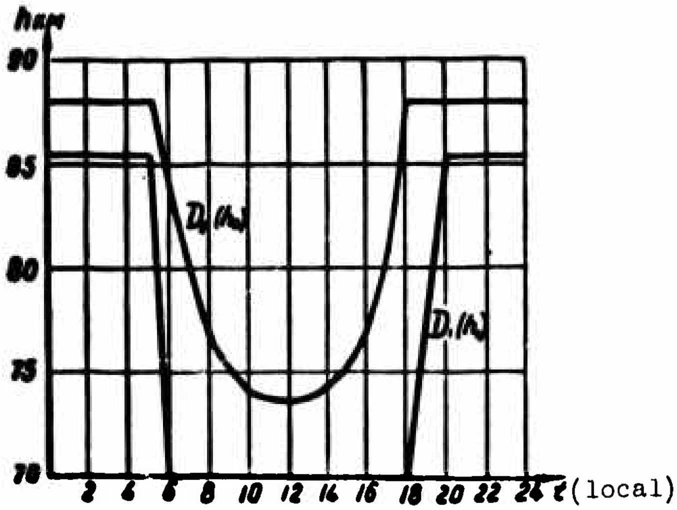


Fig. 9.

For nighttime (Fig. 8b) we do not have reliable graphs of dependence of field E on distance similar to Fig. 4b, but it is possible to assume that two-stage picture of ionization depicted in Fig. 8b is result of idealization of problem and that in reality $\omega_0^2(h)$ is characterized by smooth drop, the "trail" of ionization of layer E at night. It is interesting to note that jumps of ω_0^2 on boundary h_1 by day and night are approximately identical.

Data on daily variation of phase (Figs. 5 and 6) permit us to find daily variation of heights h_1 and h_2, depicted in Fig. 9. As can be seen from Fig. 9, lower bound of layer of ionosphere h_1 follows geometric shadow of earth up to night layer E_1. Second layer h_2 is connected with height of sun χ_c by empirically found dependence

$$h = h_0 + A \ln \sec \chi_c, \tag{3.10}$$

where $h_0 \cong 70$ km and A is slowly changed during the course of the year from 3 to 7 [31].

Formula (3.10) is valid to $\chi_c = 85^\circ$. For distant field, as will be seen later, basic role is played by lower layer $h = h_1$.

CHAPTER IV

BASIC CHARACTERISTICS OF NORMAL WAVES OF FIRST NUMBERS

§ 1. Criteria of Similarity

Having completed calculations made in Chapter III, we know all parameters of medium. Therefore it is possible to turn to last operation of solution of mixed problem per diagram of Fig. 1, calculation of distant [VLW] (СДВ) field by spectral formulas (2.101)-(2.108) (model B) and (2.127), (2.128) (model C). In these spectral formulas enter following characteristics of normal waves:

1) wave numbers $\nu_j = \alpha_j + i\beta_j$,

2) coefficients of excitation $n_j = \lambda\sqrt{\frac{\pi}{N_j}}\nu_j^{\frac{1}{2}}$, (4.1)

3) polarization factors

$$\varkappa_j = \frac{Z_j(r)}{Y_j(r)} \cdot \frac{D_{\nu_j}(a',r) - \tilde{Z}_y D_{\nu_j}(a,r)}{D_{\nu_j}(a,r) - \frac{1}{\tilde{Z}_s} D_{\nu_j}(a',r)}, \tag{4.2}$$

4) forms of normal waves $Y_j(r)$.

In model B they are constant, and in model C the change slowly, depending upon geographic coordinates θ and φ. Calculation of ν_j, n_j, $\varkappa_j$ and Y for both models is reduced to finding eigenvalues of χ_j and eigenvectors $|Y_j|Z_j|$ of operator of normal waves, described by expressions (2.53)-(2.57) for particular case of four-layer medium k = 0, 1, 2, 3, where layer k = 0 is earth, k = 1 is atmosphere, and k = 2 and k = 3 are layers of ionosphere.

Thus, ν_j, n_j, $\varkappa_j$, Y_j are functions of the following 9 dimensionless parameters, characterizing medium at given point θ, φ:

1) radius of earth $k_1a = k_1r_0$, where $k_1 = \frac{\omega}{c}$,

2) dielectric constant of earth $\varepsilon_0 = \varepsilon_0' + i\varepsilon_0''$,

3) height lower boundary of first layer $k_1h_1 = k_1(r_1 - r_0)$,

4) square of critical frequency of first layer $\omega^2_{0,1}/\omega^2$,

5) number of collisions in first layer $\nu^{(1)}_{eff}/\omega$,

6) height of lower boundary of second layer $k_1h_2 = k_1(r_2 - r_0)$,

7) square of critical frequency of second layer $\omega^2_{0,2}/\omega^2$,

8) number of collisions in second layer $\nu^{(2)}_{eff}/\omega$,

9) gyromagnetic frequency ω_H/ω.

From expressions for operator of normal waves (2.53)-(2.57), and also (4.1) and (4.2), it follows that ν_j, n_j, $\varkappa_j$ and Y_j are analytic functions of these parameters. Contemporary mathematical means do not permit us to find these functions in analytic form; therefore we used ordinary, well approbated, direct mathematical methods of finding eigenvalues and functions, which give numerical values of $\varkappa_j$ and $|Y_j(r)/Z_j(r)|$ for given numerical values of parameters of medium k_1a, ε_0; k_1h_1; k_1h_2, etc. Eigenvalues of χ_j, and this means of ν_j also, were calculated with accuracy to 5 significant digits, while $Y_j(r)$ and $Z_j(r)$ were calculated to 3 significant digits. Coefficients of excitation and polarization factors were found from calculated ν_j and $|Y_j/Z_j|$ on the basis of formula (4.1) and (4.2) with accuracy of 3 significant digits.

Exact calculation of ν_j, n_j, $\varkappa_j$ and Y_j over wide range of changes of nine shown parameters of medium by direct algebraic methods involves a great deal of work, which scarcely need be done for clarification of basic regularities of distant VLW field. For reduction of volume of calculation operations we adhered to the following, which were checked by calculations:

1. At distances of more than 2000 km basic contribution to spectral formulas of models B and C in range of frequencies of 10-20 kilocycles is made by first three normal waves, called quasi-TH_1, quasi-TH_2, and quasi-TE_1. Remaining types of normal waves may be omitted from calculation.

2. For shown types of normal waves basic role in waveguide process is played by lower bound of first layer of ionosphere $h_1 = r_1 - r_0$ and surface of earth, whereas for high types of waves the role of waveguide is played by boundary of second layer of ionosphere and surface of earth (see Fig. 11). Therefore characteristic criteria of similarity for first types of waves will be:

$$k_1a;\ \varepsilon_0 = \varepsilon'_0 + i\varepsilon''_0;\ k_1h_1;\ \frac{\omega^2_{0,1}}{\omega^2};\ \frac{\nu^{(1)}_{eff}}{\omega};\ \frac{\omega_H}{\omega}.$$

3. For frequencies in 10-20 kilocycle range number of characteristic criteria can be further reduced, since

a) in virtue of high conductivity of soil and sea water $\varepsilon'_0 << \varepsilon''_0 = \frac{4\pi\sigma_0}{\omega}$ and ε'_0 is immaterial

b) under the condition that frequency $\omega(\sim 10^5)$ is almost one order of magnitude less than $\nu_{eff}(\sim 10^6)$, components of tensor $\|\varepsilon\|$ depend only on two parameters $\frac{\omega_r}{\omega}$ and τ

$$\varepsilon_{rr} \simeq 1 + i\frac{\omega_r}{\omega\cos\tau},$$

$$\varepsilon_{\theta\theta} = \varepsilon_{\varphi\varphi} \simeq 1 + i\frac{\omega_r}{\omega}\cos\tau,$$

$$\varepsilon_{\theta\varphi} = -\varepsilon_{\varphi\theta} = -i\frac{\omega_r}{\omega}\sin\tau,$$

where

$$\frac{\omega_r}{\omega} = \frac{\omega_0^2}{\omega\sqrt{\nu_{eff}^2 + \omega_H^2}} = \frac{\omega_0^2}{\omega\nu_{eff}}\cos\tau,$$

$$\tau = \operatorname{arctg}\frac{\omega_H}{\nu_{eff}}.$$

Thus in working range of frequencies ω wave numbers ν_j are functions of five criteria:

$$\nu_j = \nu_j\left(k_1 a;\ \frac{4\pi\sigma_0}{\omega};\ k_1 h_1,\ \frac{\omega_{r1}}{\omega},\ \tau_1\right),$$

here $\omega_{r,1}/\omega$ and τ_1 pertain to lower layer of ionosphere. On these criteria of similarity of medium also depends remaining characteristics of normal waves n_j, $\varkappa_j$, and Y_j.

For study of field of VLW at the surface of earth, where $Y_j = 1$, it is necessary to know only three characteristics of normal waves ν_j, n_j, and $\varkappa_j$, as functions of 5 above-indicated criteria of similarity of medium in ranges corresponding to possible values of parameters of medium on route at different times of day and year.

Before describing results obtained, let us divide discrete spectrum of normal waves into two series of normal waves, called quasi-TH_j ($j = 0, 1, 2, 3, \ldots$ and quasi-TE_j ($j = 1, 2, 3, \ldots$). By definition, first types of waves, for $\tau \to 0$, continuously pass into ordinary waves of type TH_j ($j = 0, 1, 2, \ldots$) with components H_φ, E_r, E_θ, while second become ordinary TE_j waves with components E_φ, H_r and H_θ.

TH_j and TE_j waves determine electromagnetic field in Watson model, where magnetic field of earth is absent ($\tau = 0$). Here vertical Hertz doublet excites only TH-waves.

In case of $\tau \neq 0$ waves of quasi-TH_j type with wave numbers ν_{jH} ($j = 0, 1, 2, 3, \ldots$), besides basic components H_φ, E_r and E_θ, contain components E_φ, H_r, and H_θ. Ratio of two groups of components is determined by magnitude of polarization factor $\varkappa_{jH}$. Analogously, quasi-TE_j waves with wave numbers ν_{jE}, besides basic components E_φ, H_r, and H_θ, contain components H_φ, E_r, and E_θ in ratio determined by quantity $1/\varkappa_{jE}$. With sufficiently large τ, $\varkappa_{jE}$ and $\varkappa_{jH}$ become comparable to 1; therefore criterion of domination of components E_φ, H_r, and H_θ in quasi-TE_j waves and of components H_φ, E_r, E_θ in quasi-TH_j waves cannot be used for discerning TH_j and TE_j waves. For their identification during calculations we continuously decreased τ and 0.

It is natural that when $\tau \neq 0$, vertical Hertz doublet excites both quasi-TH_j and quasi-TE_j waves. Coefficients of excitation of these waves subsequently are designated n_{jH} and n_{jE} respectively.

§ 2. Dependence of Characteristics of Normal Waves on Wave Number

In Fig. 10 are depicted spectra of normal waves of type TH_j for $j = 0, 1, 2, 3, 4, 5$ and 6 for following parameters of medium: $a = 6370$ km, $\sigma_0 = \infty$, $h_1 = 70$ km, $h_2 = 75$ km, $\frac{\omega_{r,1}}{\omega} = 0.15$; $\frac{\omega_{r,2}}{\omega} = 1$; tg $\tau_1 = 0.4$; tg $\tau_2 = 0.8$, and $\omega = 10^5$, which corresponds to daytime conditions of propagation above marine surface. On abscissa are plotted $\alpha_j = \mathrm{Re}(\nu_j)$, while ordinate are plotted, upwards, mod n_j and, downwards, $\beta_j = \mathrm{Im}(\nu_j)$.

In Fig. 11 are depicted distributions of amplitude and phase φ of component E_{rj} with respect to height for $j = 1, 2$, and 4 (TH_1, TH_2, and TH_4).

On the basis of Figs. 10 and 11 the following conclusions can be made.

1) Real parts of wave numbers $\alpha_j = \mathrm{Re}(\nu_j)$ monotonically decrease with growth of wave number. The biggest value of α_0 for TH_0 wave is always smaller than magnitude of $k_1 r_1$, which is due to positive value of angle of slip γ with incidence of normal waves on layer of ionosphere:

$$\cos \gamma_j \approx \mathrm{Re}(\nu_j)/k_1 r_1. \tag{4.3}$$

2) Period of modulation of amplitude of normal wave (Fig. 11) with respect to front decreases with increase of wave number j.

3) Coefficients of excitation n_j of normal waves sharply decrease for first types of waves, for which $\alpha_j \geq k_1 a$, which is caused by low field strength of these waves near earth's surface (adhesion effect).

4) Attenuation factors $\beta_j = \text{Im}(\nu_j)$ of normal waves monotonically increase with increase of number j. Exception is TH_0 wave, which experiences large degree of damping. TH_0 wave strikes ionosphere at an angle close to quasi-Brewster angle; therefore it is reflected weakly from ionosphere.

5) From Fig. 11 one may see that first types of waves are reflected basically from lower boundary of first layer, and high types of waves, starting with TH_4, are reflected from second layer.

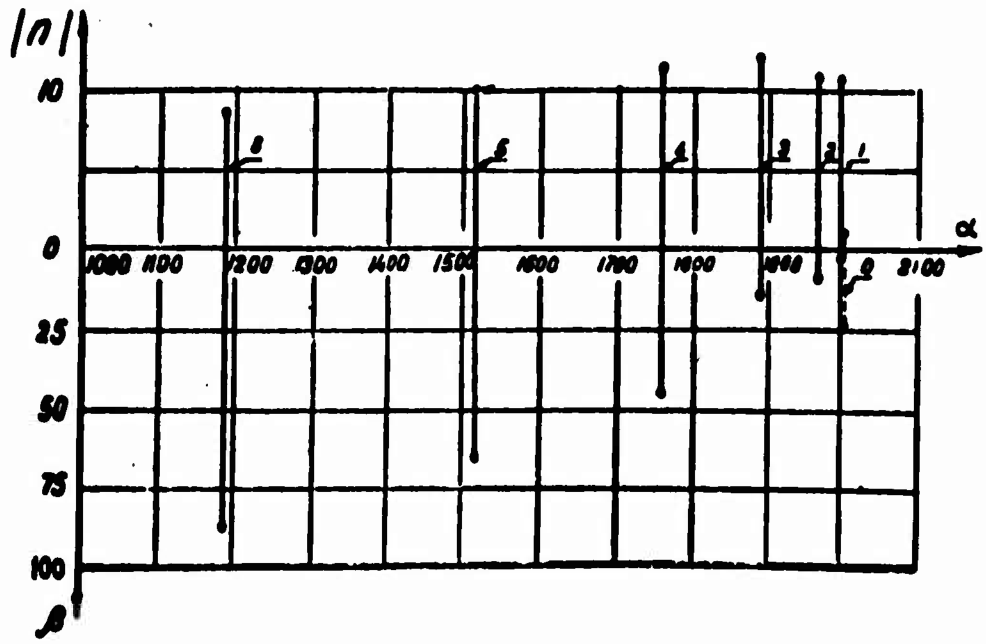


Fig. 10.

Numerical values of n_j are multiplied by $2\sqrt{2}\cdot 10^{10}$.

Of special interest is TH_0 wave, which is analog of cable wave in two-conductor waveguide systems. In flat models of media it is considered the basic wave [27, 28, 29]. During calculation of sphericity of earth, cable wave loses dominating value, because its field is concentrated near concave surface of ionosphere and is weakened greatly near earth (adhesion effect); coefficient of excitation $n_0 << n_j$ $(j \neq 0)$. Besides, in daytime cable wave possesses experiences greater degree of damping from incidence at angle close to Brewster angle.

From Figs. 10 and 11 it follows that in daytime at distances over 1000 km it is necessary to consider only quasi-TH_1 and quasi-TH_2 waves.

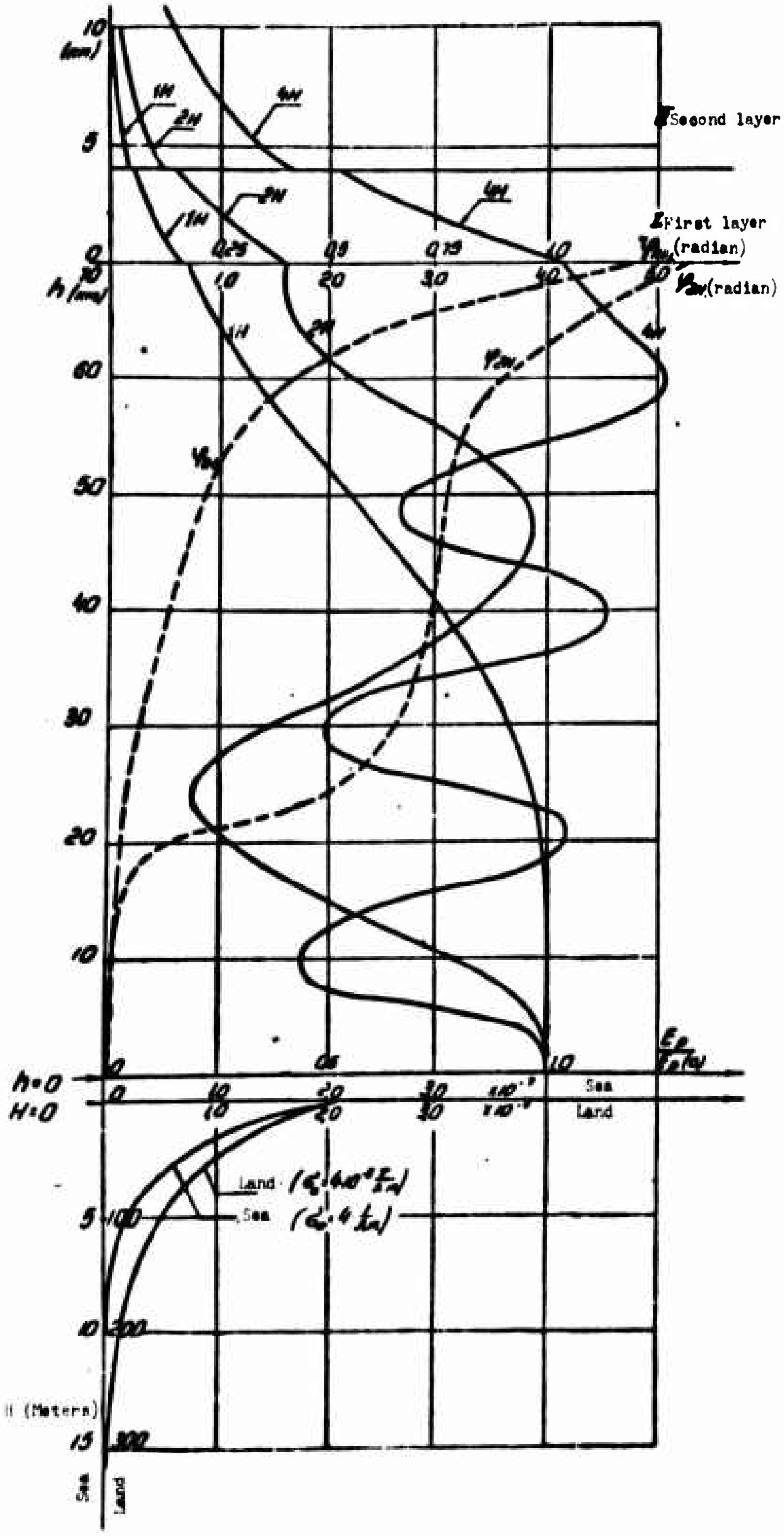


Fig. 11.

For nighttime, dependencies of ν_j, n_j, $\varkappa_j$ and Y_j on number j have analogous form. Number of propagating normal waves of quasi-TH type is increased to 8; therefore $\Delta\alpha_j = \alpha_j - \alpha_{j-1}$ decrease. Besides, all α_j are displaced to the right; therefore larger number of first types of normal waves is subjected to adhesion effect, showing up as decrease of coefficients of excitation n_j. At night qausi-TE_j waves start to play a role. Their wave numbers $\alpha_{jE} = \mathrm{Re}\,(\nu_{jE})(j = 1, 2, 3, \ldots)$ also monotonically decrease with growth of number j, starting from first TE_1 wave, and attenuation factors monotonically increase.

Increase of excitation frequency ω leads to change of spectra (Fig. 10), to analogous increase of height of layer h_1. Number of TH_j and TE_j waves is increased, that is, $\Delta\alpha_j$ decrease, and all α_j are displaced to the right. Therefore with increase of ω, an ever greater number of first types of normal waves falls in interval $k_1 a < \alpha < k_1(a + h_1)$, where adhesion effect appears. Thus they cease playing role in spectrum of distant field, and their place is taken by higher types of waves.

Calculation shows that in range of frequencies from 10 to 20 kilocycles and with change of height of layer h_1 from 70 to 90 km at distances of more than 2000 km, as a rule, basic role in distant field is played at night by three waves: quasi-TH_1, quasi-TH_2, and quasi-TE_1. Exception to this occurs in certain sections of route, where, as a result of interference, there occurs mutual extinguishing of quasi-TH_1 and quasi-TH_2 waves ($\theta = \theta_{\min\min}$, Chapter V, § 2). At these singular points it is necessary to consider higher types of waves.

Subsequently we shall limit consideration to but three types of normal waves, TH_1, TH_2, and TE_1, since above-indicated sections of route are unfit for communication and navigation.

§ 3. Dependence of Characteristics of Normal Waves on Properties of Ionosphere

Properties of ionosphere are determined by three criteria of similarity $k_1 h_1$, $\omega_{r,1}/\omega$ and τ_1. In Figs. 12, 13, and 14 is given complex plane $\nu = \alpha + i\beta$, on which are plotted curves of ν_{1H}, ν_{2H}, and ν_{1E} as functions of parameter $\omega_{r,1}/\omega$ of first layer of ionosphere for two different values of parameters τ and $k_1 h$, with $\omega = 10^5$:

a) $$\mathrm{tg}\,\tau_1 = 0{,}4\ (\nu_{\text{eff}}^{(1)} = 2.10^7),\ k_1 h_1 = 22\ (h_1 = 70\ \text{км}),\ h_2 = 75\ \text{км},\ \nu_{\text{eff}}^{(2)} = 10^7,$$

$$\frac{\omega_{r,2}}{\omega} = 1;$$

b) $$\mathrm{tg}\,\tau_1 = 0{,}4\ (\nu_{eff}^{(1)} = 2.10^7),\ k_1 h_1 = 28\ (h_1 = 88{,}5_{км}),\ h_2 = 93_{км},\ \nu_{eff}^{(2)} = 10^7,$$
$$\frac{\omega_{r,2}}{\omega} = 1;$$

c) $$\mathrm{tg}\,\tau = 10\ (\nu_{eff}^{(1)} = 8.10^5),\ k_1 h_1 = 22\ (h_1 = 70_{км}),\ h_2 = 75_{км},\ \nu_{eff}^{(2)} = 5.10^6,$$
$$\frac{\omega_{r,2}}{\omega} = 1{,}5;$$

d) $$\mathrm{tg}\,\tau = 10\ (\nu_{eff}^{(1)} = 8.10^6),\ k_1 h_1 = 28\ (h_1 = 88{,}5_{км}),\ h_2 = 93_{км},\ \nu_{eff}^{(2)} = 5.10^5,$$
$$\omega_{r,2}/\omega = 1{,}5.$$

Remaining parameters of medium had values $k_1 a = 2001.2$; $\sigma_0 = \infty$.

Increase of $\omega_{r,1}/\omega$ along every curve (Figs. 12-14) occurs because of increase of electron concentration $N_e^{(1)}$ of first layer of ionosphere, that is, of quantity $\omega_{0,1}^2$ (In Figs. 12-14 are plotted values of $\frac{\omega_{r,1}}{\omega}$).

Real parts of ν_{1H}, ν_{2H}, and ν_{1E} monotonically grow with decrease of ω_r/ω.

Changes of dampings β_{1H}, β_{2H}, and β_{1E} as functions of ω_r/ω are more complicated. The most simple is dependence of β_{1E} on ω_r/ω for quasi-$\mathbf{TE}_1$ wave (Fig. 14). With decrease of $\frac{\omega_r}{\omega}$ from ∞ (ideally conducting ionosphere), damping $\beta_{1E} = \mathbf{Im}\ (\nu_{1E})$ monotonically increases from 0. Here real part of wave number $\alpha_{1E} = \mathrm{Re}\ (\nu_{1E})$ is increased proportionately, so that line $\nu_{1E}\left(\frac{\omega_r}{\omega}\right)$ becomes straight. Increase of β_{1E} is explained by weakening of reflective properties of h_1 layer of ionosphere.

For $\mathbf{TH}_1$ and $\mathbf{TH}_2$ waves monotonic build-up of β_{jH} with decrease of ω_r/ω is disturbed in range of values of ω_r/ω approximately from 10 to 0.5, where there is observed a quite sharp fall β_{1H} and β_{2H} with decrease of ω_r/ω.

When ω_r/ω = 10 to 20, maximum damping occurs. Appearance of maximum damping is due to fact that normal waves, quasi-$\mathbf{TH}_1$ and quasi-$\mathbf{TH}_2$, with predominance of vertical polarization ($\varkappa_{1H}$ and $\varkappa_{2H}$ are small), strike layer of ionosphere $\mathbf{r} = \mathbf{r}_1$ at angles close to quasi-Brewster for values of $\omega_r/\omega \cong$ 10 to 20. Therefore in this range of values of ω_r/ω reflective qualities of ionosphere are weakened. Subsequently, for brevity, we shall call this phenomenon the Brewster effect.

When $\frac{\omega_{r,1}}{\omega}$'s are considerably smaller than quasi-Brewster values, there is restored normal increase of β_{jH} with decrease of $\frac{\omega_{r,1}}{\omega}$, caused by weakening of reflective qualities of ionosphere. Between this normal region and region of quasi-Brewster maximum, in interval $5 > \frac{\omega_{r,1}}{\omega} > 0.3$, damping is minimum, wherein lie

values of ν_{1H} and ν_{2H} corresponding to normal conditions of lower layers of ionosphere.

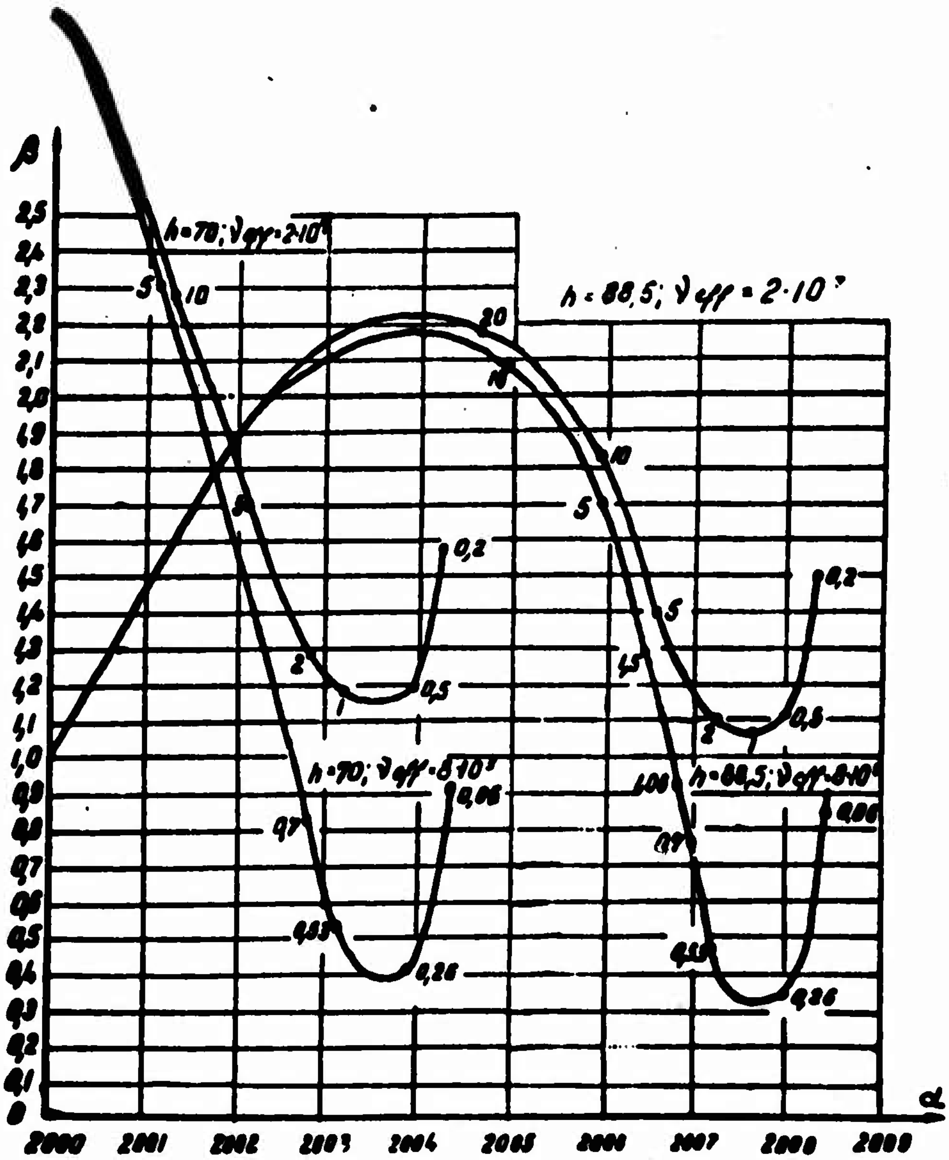


Fig. 12.

For values of $\omega_{r,1}/\omega$ less than 0.2 lower layer of ionosphere is so greatly weakened that basic role is played by higher disposed second layer of ionosphere.

From comparison of curves (Figs. 12-14), with tg τ = 0.4 and tg τ = 10, it follows that magnetic field of earth starts to show up with large values of τ, when ν_{eff} is less than ω_H. Here attenuation factors decrease. Influence of magnetic field of earth is greatest for wave quasi-TH_1 waves, less for quasi-TH_2 waves, and practically negligible for quasi-TE_1 waves. Decrease of β_{1H} and β_{2H} due to magnetic field of earth is observed over wide range of value of $\omega_{r1}/\omega < 5$ and is explained by

weakening of effect of Brewster angle as a result of appearance of magnetic anisotropy of layer. In region of large values of $\omega_{r1}/\omega > 5$, where Brewster effect is maximum, influence of magnetic anisotropy is small.

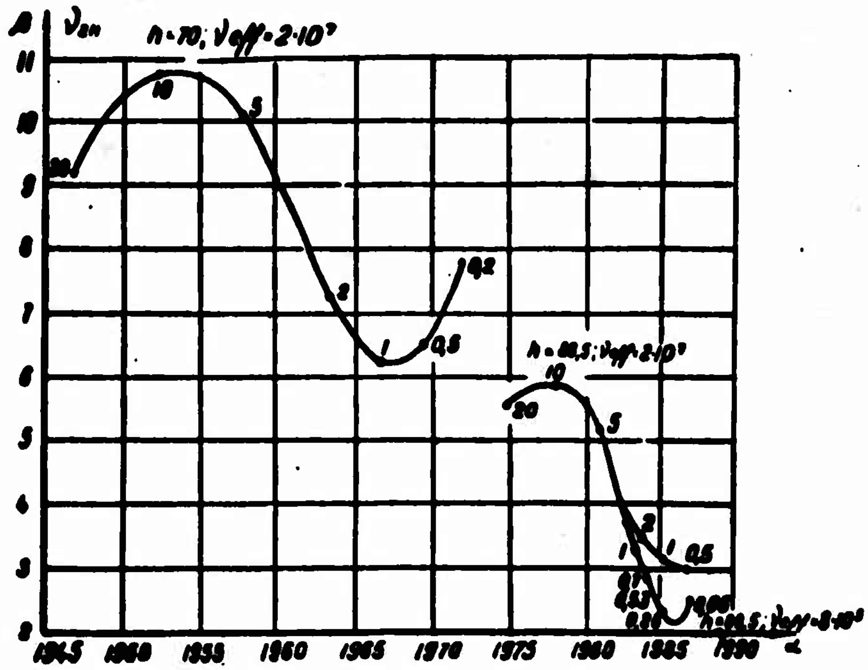


Fig. 13.

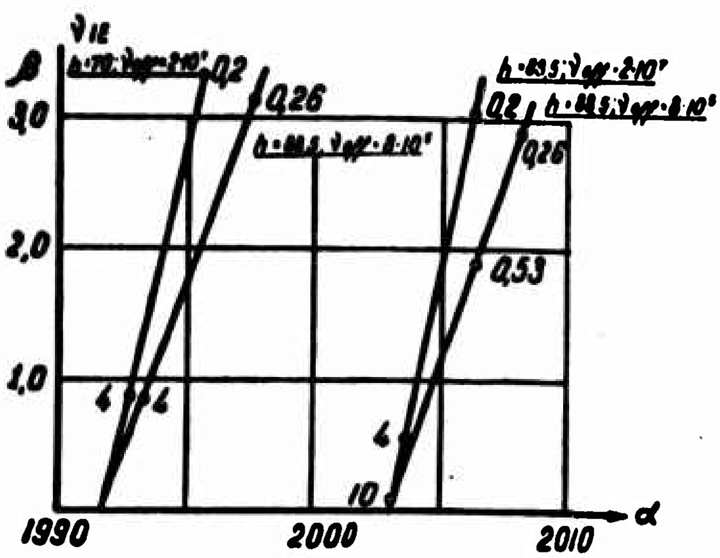


Fig. 14.

From Figs. 12-14 can be made also conclusions about altitude effect of ionosphere on Re (ν_j) and Jm (ν_j). When $\frac{\omega_r}{\omega} > 0.2\text{-}0.3$, real parts of wave numbers α_{1H}, α_{2H}, and α_{1E} grow with h_1, just as for all other α_j (Fig. 10). Here speed of displacement $\frac{\partial \alpha_{1H}}{\partial h}$ is least; therefore intervals between α_{1H} and α_{2H}, and also between

α_{1H} and α_{1E}, decrease with growth of h. This regularity, shown earlier during discussion of spectra (Fig. 10), very significantly changes picture of distant field during transition from day to night. Thickening of spectrum with increase of width of waveguide h_1, or of excitation frequency ω (criterion k_1h_1), is general dependence for all hollow waveguide systems.

Attenuation factors β_{1H}, β_{2H}, β_{1E} drop with increase of h.

§ 4. Dependence of Characteristics of Normal Waves on Properties of Earth

Properties of earth are determined by parameters k_1a and $\frac{4\pi\sigma_0}{\omega}$. Influence of earth's curvature on wave numbers in range $r_0 \cong 6350$-6380 km is well within following linear dependence:

$$\frac{a_j(r_0)}{r_0} = \frac{a_j(a)}{a}, \tag{4.4}$$

where a = 6370 km.

For high types of waves this dependence is satisfied in wider limits than for first types of waves, where this dependence is distorted by effect of "adhesion." With increase of radius of earth r_0, effect of "adhesion" is less, and formula (4.4) is satisfied with large degree of accuracy. Conversion of α_j from plane case ($a \to \infty$) to spherical with a = 6370 km by formula (4.4) for first types of waves is impermissible, which indicates unfitness of flat models for calculation of distant field of VLW. When a = 6370 km, form of TH_1 wave, determining distant field, differs considerably from TH_1 wave of flat case ($a = \infty$).

In range of radii of earth of 6350-6380 km it is possible to consider that forms of waves, coefficients of excitation n_j, damping β_j, and polarization $\varkappa_j$ remain constant under the condition that other parameters of medium k_1r_1, $\frac{\omega_r}{\omega}$, τ etc., are not changed.

More essential influence on characteristics of normal waves is rendered by conductivity of "soil." In Fig. 15 is depicted dependence of attenuation factors of quasi-TH_1 and quasi-TH_2 waves on resistance $1/\sigma_0$ for day (h_1 = 70 km, $\nu_{eff}^{(1)} = 2.10^7$, etc.) and night (h = 88.5 km, $\nu_{eff}^{1} = 8.10^5$, etc.) cases when $\omega = 10^5$. As can be seen from graph, in range of "soil" resistances from sea water to very dry soil attenuation factor of TH_1 wave is increased by 0.7, while that of TH_2 wave is increased by 0.9-1.0.

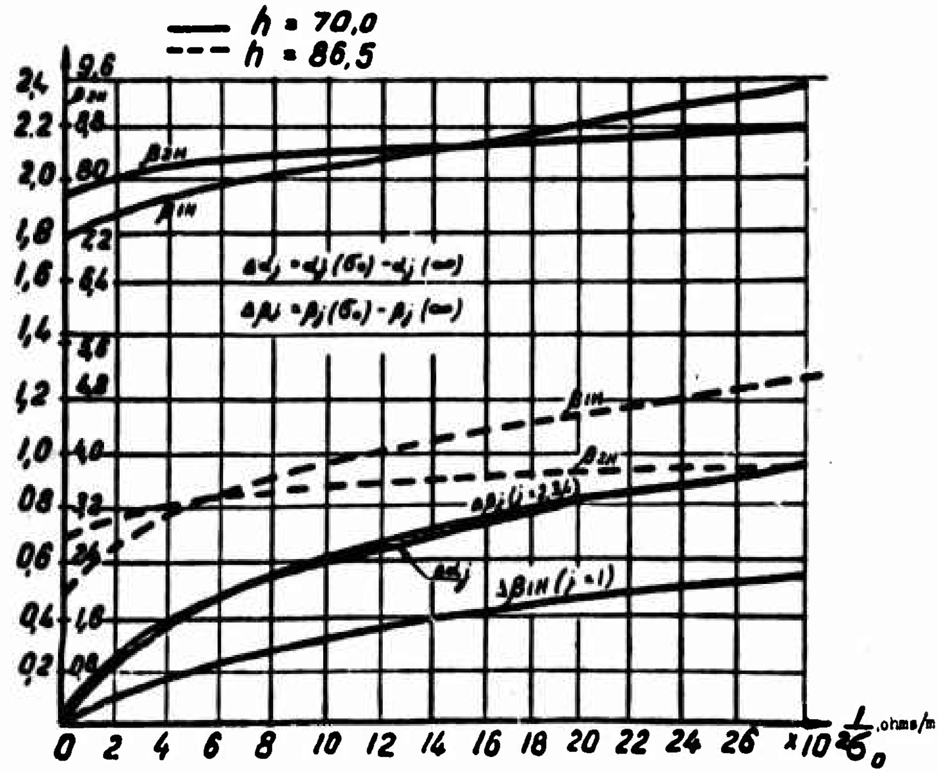


Fig. 15.

For approximate calculation of influence of conductivity for all types of waves it is possible to use formulas:

$$\beta_j(\sigma_0) = \beta_j(\infty) + \Delta\beta_j(\sigma_0),$$

$$\alpha_j(\sigma_0) = \alpha_j(\infty) + \Delta\alpha_j(\sigma_0),$$

where $\nu_j(\infty) = \alpha_j(\infty) + i\beta_j(\infty)$ is wave number of normal wave j for $\sigma_0 = \infty$; quantities $\Delta\alpha_j(\sigma_0)$ and $\Delta\beta_j(\sigma_0)$ are presented in Fig. 15; $\Delta\beta_{1H}$ for TH_1 wave is plotted separately, since influence of σ_0 on TH_1 wave is somewhat less than for high types of waves, owing to effect of adhesion.

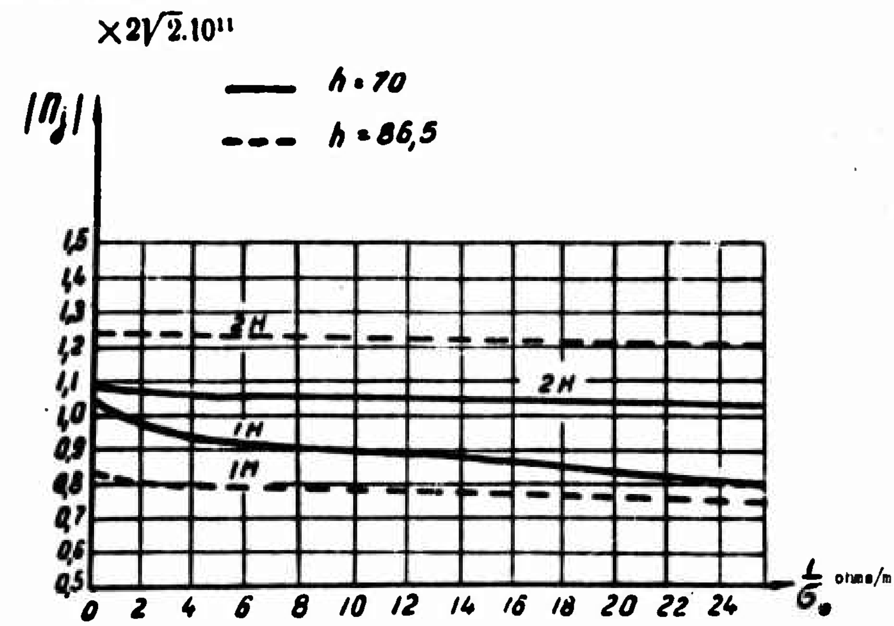


Fig. 16.

On the same Fig. 15 is given dependence of increase of $\Delta\alpha_j$ with impairment of ground conductivity. $\Delta\alpha_{1H}$ is small and is not shown in Fig. 15.

In Fig. 16 is given dependence of coefficients of excitation n_j for quasi-TH_1 and quasi-TH_2 waves on resistivity of soil $1/\sigma_0$. As can be seen from graphs of Fig. 16, influence of $1/\sigma_0$ on n_j is insignificant. Just as insignificant is influence of resistance of soil on coefficients of polarization $\varkappa_j$.

§ 5. Dependence of Characteristics of Normal Waves on Parameters of Radiator

Radiating device in the form of vertical Hertz doublet is characterized by emissive power W, coordinates of dipole, and frequency ω.

Emissive power, as follows from formulas (2.101) and (2.103), determines amplitudes of normal waves, which are proportional to square root of emissive power. Characteristics of normal waves ν_j, $\varkappa_j$, n_j, and γ_j do not depend on emissive power.

Position of dipole, determined by height of dipole above earth $b - r_0$, does not affect ν_j and $\varkappa_j$, but can strongly change coefficients of excitation $n_j(0)$ at point 0.

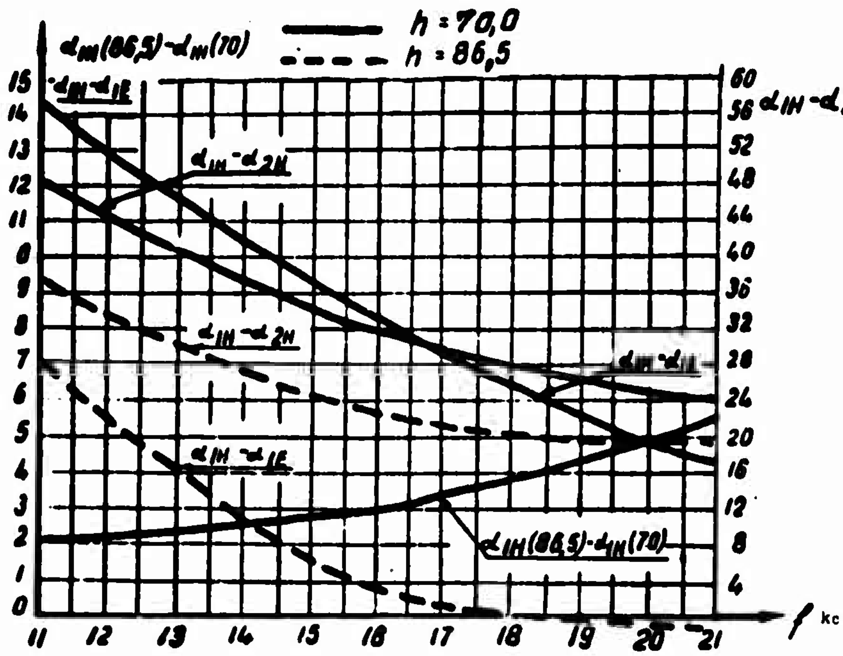


Fig. 17.

Frequency ω of radiator essentially affects all characteristics of normal waves, since it enters in characteristic criteria k_1a, $4\pi\sigma_0/\omega$, ω_r/ω, and k_1h_1.

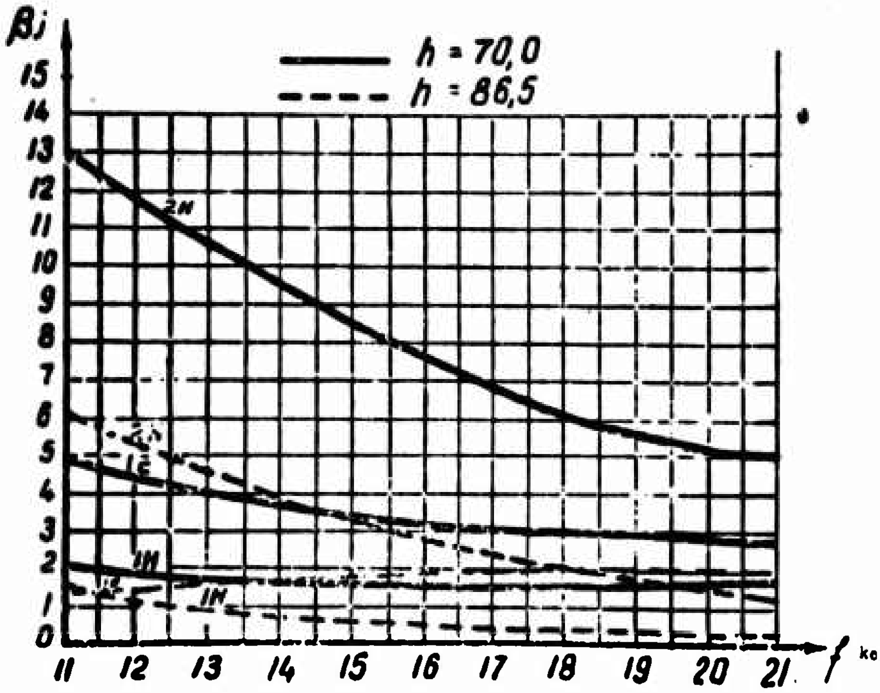


Fig. 18.

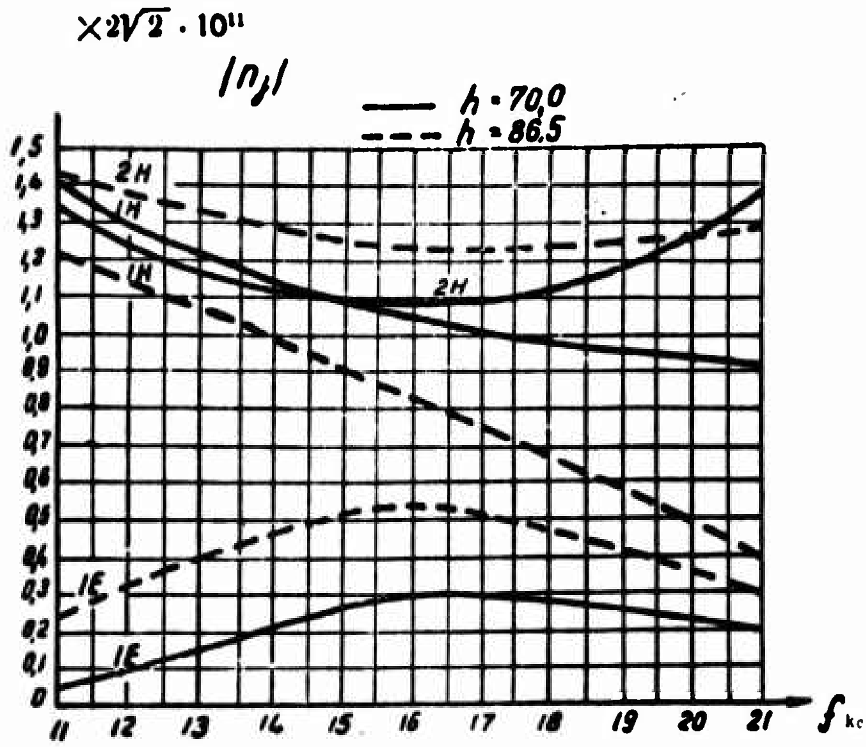


Fig. 19.

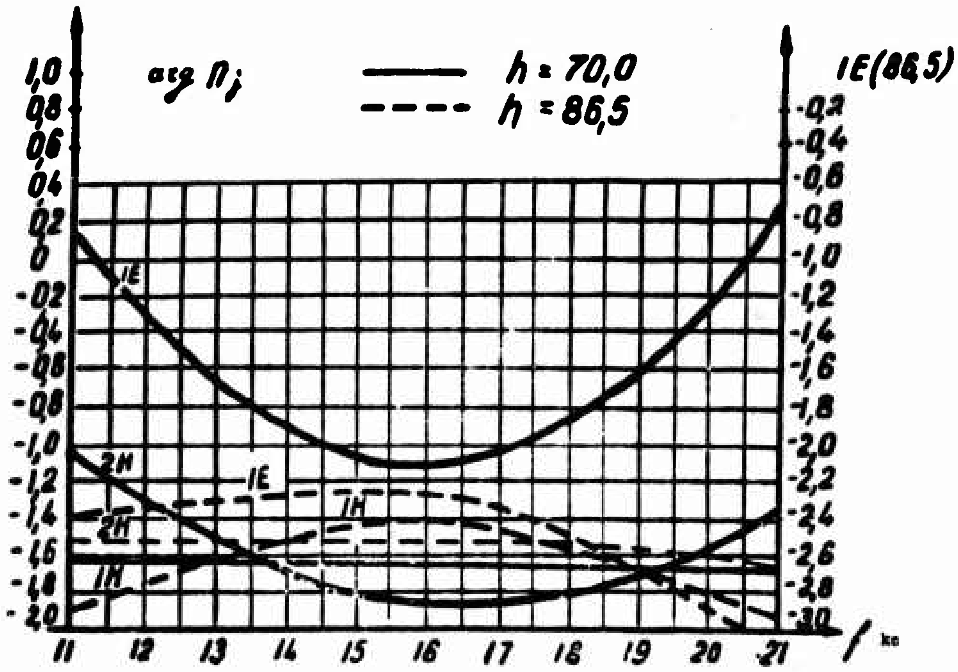

arg n_j
h = 70,0
h = 86,5
IE(86,5)
IE
2H
IH
f kc

Fig. 20.

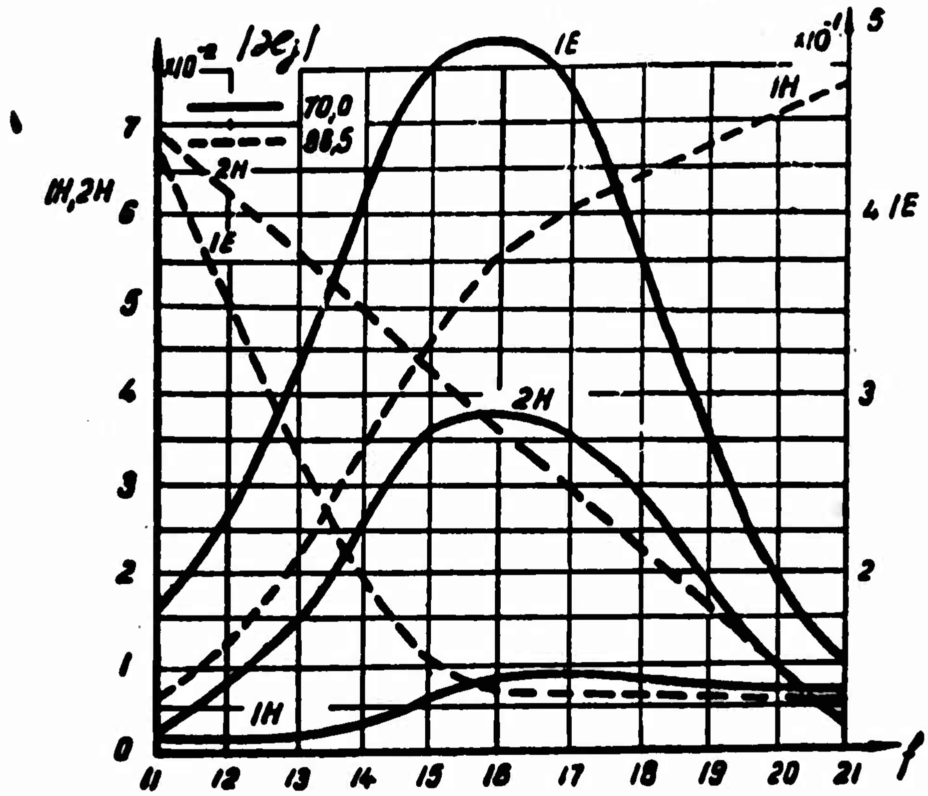

$|\varkappa_j|$
x10⁻²
x10⁻¹
70,0
86,5
IH,2H
IE
2H
IH
f

Fig. 21.

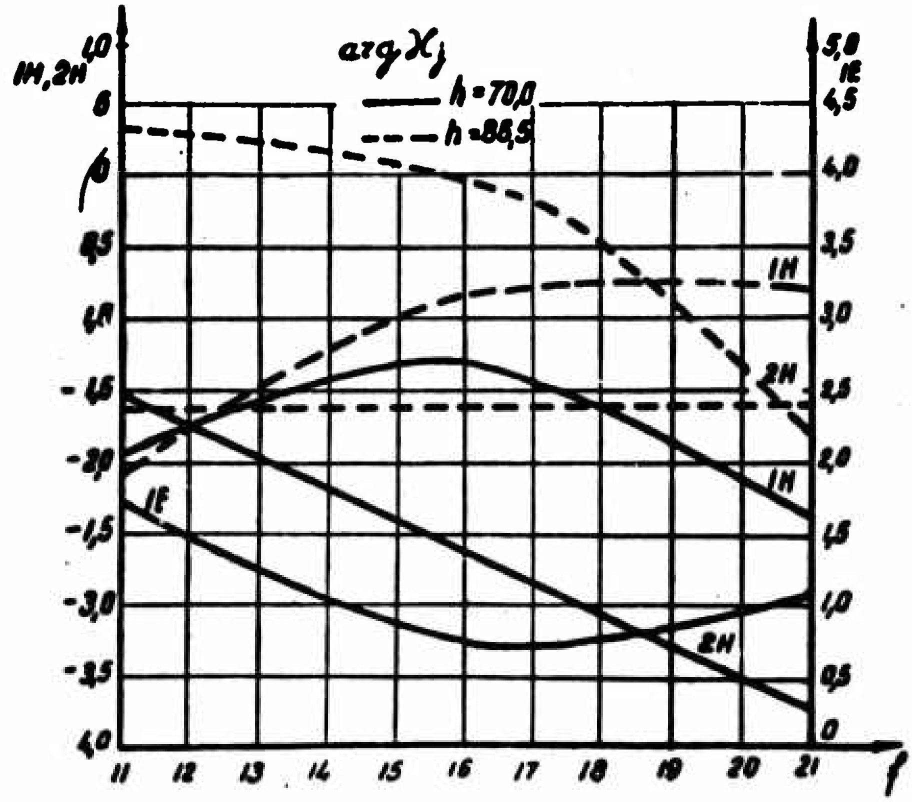


Fig. 22.

General character of regularities of change α_j and β_j for TH_1, TH_2, and TE_1 waves from frequency ω may be established from Figs. 12-14. With increase of ω, criterion k_1h_1 is increased, while ω_r/ω decreases, where, according to Figs. 12-14, all α_j increase and β_{1H} and β_{2H} decrease, if ω_r/ω lies in "tail" of quasi-Brewster maximum and increase for region of small ω_r/ω. Since influence of frequency ω on characteristics of normal waves is complicated effect, in which participate 4 parameters of medium, then for convenience of calculation in Figs. 17-22 are given to dependencies of the following quantities on frequency:

$$(\alpha_{1H}-\alpha_{2H});(\alpha_{1H}-\alpha_{1E});\beta_j;\ \mathrm{mod}\, n_j;\ \arg n_j;\ \mathrm{mod}\, \varkappa_j;\ \arg \varkappa_j.$$

On every figure are given two curves for "day" and "night" conditions. They are basic working graphs for calculations of amplitude and phase of distant field (Chapter V). For "day" curves are taken parameters of medium

$$\varepsilon_0=\infty,\ a=6370\ \text{км},\ h_1=69{,}73\ \text{км}:\ \omega^2_{0,1}=0{,}3\cdot10^{12},\ \nu^{(1)}_{eff}=2\cdot10^7,$$

$$h_2=75\ \text{км},\ \omega^2_{0,2}=2\cdot10^{12},\ \nu^{(2)}_{eff}=10^7,\ \omega_H=8\cdot10^6.$$

For "night" curves are taken parameters of medium

$$\varepsilon_0=\infty,\ a=6370\ \text{км},\ h_1=86{,}5\ \text{км},\ \omega^2_{0,1}=0{,}3\cdot10^{12};$$

$$\nu^{(1)}_{eff}=8\cdot10^5,\ h_2=90\ \text{км},\ \omega^2_{0,2}=1{,}5\cdot10^{12},\ \nu^{(2)}_{eff}=5\cdot10^5,\ \omega_H=8\cdot10^6.$$

In addition, in Fig. 17 is given dependence $\Delta\alpha_{1H} = \alpha_{1H}$ (86.5) - α_{1H} (70). Magnitudes of α_{2H} and α_{1E} are calculated by the formulas: α_{2H} (70) = α_{1H} (70) - - (α_{1H} - α_{2H}) α_{2H} (86.5) = α_{1H} (70) + $\Delta\alpha_{1H}$ - (α_{1H} - α_{2H}), etc, where α_{1H} (70) = = 1611.4; 2004.0; 2604.4, respectively for ω = 0.805; 1.000; 1.290 $\times$ $0.942 \cdot 10^5$.

Parameters of waves for intermediate heights are interpolated per linear law.

For calculation of conductivity of earth, Figs. 17-19 for α_j, β_j and $|n_j|$ are corrected by graphs of Figs. 15 and 16. Phases of n_j and $\varkappa_j$ weakly depend on conductivity of earth.

Deviation of magnitudes of α_j and β_j from values in Figs. 17-19 due to change of $\omega_{r,1}/\omega$ and τ, which can take place in periods of ionospheric perturbations (flashes on sun), is determined with help of graphs of Figs. 12-14.

Thus with help of basic working graphs of Figs. 17-22, corrected by graphs of Figs. 12-14, allowing for deviations of ω_r/ω and τ, graphs of Figs. 15-16 for calculation of conductivity σ_0, formula (4.4) for calculation of changes of earth radius a, and formula (2.98) for calculation of heights of points of radiation and reception, it is possible to calculate all characteristics of normal waves necessary for calculation of distant field.

CHAPTER V

CALCULATION OF REGULAR DEPENDENCIES OF AMPLITUDE AND PHASE OF DISTANT VLW FIELD ON TIME AND DISTANCE

§ 1. Calculation Formulas for Components of Distant VLW Field (Model C)

As already was indicated in Chapter I, adequate theory of [VLW] (СДВ) propagation can be formulated only on the basis of model of medium considering temporal and horizontal variations of parameters of medium. Therefore we shall use subsequently calculation formulas for model C. Using formulas (2.25)-(2.30), with function $|B/A|$ determined by (2.128), we obtain following calculation formulas for components of field:

$$\left.\begin{aligned}
H_\varphi &\approx iA_0'\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} n_j(O)\, n_j(P)\,\overline{D_{\nu_j}(a',b)}\,\overline{D_{\nu_j}(a',r)}\,e^{i\int_0^\theta \nu_j d\theta+\frac{\pi i}{4}}\\
E_r &\approx -iA_0\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} n_j(O)\, n_j(P)\,\overline{D_{\nu_j}(a',b)}\,\overline{D_{\nu_j}(a',r)}\,e^{i\int_0^\theta \nu_j d\theta+\frac{\pi i}{4}}\\
E_\theta &\approx A_0\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} n_j(O)\, n_j(P)\,\overline{D_{\nu_j}(a',b)}\,\overline{D_{\nu_j}(a',r')}\,e^{i\int_0^\theta \nu_j d\theta+\frac{\pi i}{4}}
\end{aligned}\right\}\qquad(5.1')$$

$$\left.\begin{aligned}
E_\varphi &\approx iA_0\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} n_j(O)\, n_j(P)\,\varkappa_j(P)\,\overline{D_{\nu_j}(a',b)}\,\overline{D_{\nu_j}(a,r)}\,e^{i\int_0^\theta \nu_j d\theta+\frac{\pi i}{4}}\\
H_r &\approx iA_0'\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} n_j(O)\, n_j(P)\,\varkappa_j(P)\,\overline{D_{\nu_j}(a',b)}\,\overline{D_{\nu_j}(a,r)}\,e^{i\int_0^\theta \nu_j d\theta+\frac{\pi i}{4}}\\
E_\theta &\approx -A_0'\sqrt{\frac{W}{\sin\theta}}\,e^{-i\omega t}\sum_{j=0}^{\infty} n_j(O)\, n_j(P)\,\varkappa_j(P)\,\overline{D_{\nu_j}(a',b)}\,\overline{D_{\nu_j}(a,r')}\,e^{i\int_0^\theta \nu_j d\theta+\frac{\pi i}{4}}
\end{aligned}\right\}\qquad(5.1'')$$

where $n_j(O)$ designated coefficient of excitation at point of radiation O, and $n_j(P)$ is same at point of reception P.

Integration in exponential factors is carried out with respect to the shortest "beams," connecting point O with point P.

As can be seen from Chapter IV, during calculation of field in range of 10-20 kilocycle, it is necessary to take into account only three normal waves: quasi-TH_1, -TH_2, and -TE_1. Therefore for calculation of field on surface of earth formulas (5.1) take form:

$$\left.\begin{aligned} E_r &\simeq -iA_0\sqrt{\frac{W}{\sin\theta}}\, e^{i\frac{\pi}{4}}\,[TH_1+TH_2+TE_1]\,(\mu v/m),\\ H_\varphi &\simeq iA'_0\sqrt{\frac{W}{\sin\theta}}\, e^{i\frac{\pi}{4}}\,[TH_1+TH_2+TE_1]\,(\mu a/m),\\ E_\theta &\simeq \frac{ik_1}{k_0}A_0\sqrt{\frac{W}{\sin\theta}}\, e^{i\frac{\pi}{4}}\,[TH_1+TH_2+TE_1]\,(\mu v/m).\end{aligned}\right\} \quad (5.2)$$

$$\left.\begin{aligned} H_r &= \frac{k_1}{k_0}A'_0\sqrt{\frac{W}{\sin\theta}}\, e^{i\frac{\pi}{4}}\,[TH_1+TH_2+TE_1]^*\,(\mu a/m),\\ E_\varphi &= \frac{k_1}{k_0}A_0\sqrt{\frac{W}{\sin\theta}}\, e^{i\frac{\pi}{4}}\,[TH_1+TH_2+TE_1]^*\,(\mu v/m),\\ H_\theta &= -A'_0\sqrt{\frac{W}{\sin\theta}}\, e^{i\frac{\pi}{4}}\,[TH_1+TH_2+TE_1]^*\,(\mu a/m),\end{aligned}\right\} \quad (5.3)$$

where

$$[TH_1+TH_2+TE_1] = n_{1H}(O)n_{1H}(P)\,e^{i\int_0^P \nu_{1H}d\theta} + n_{2H}(O)n_{2H}(P)\,e^{i\int_0^P \nu_{2H}d\theta} + n_{1E}(O)n_{1E}(P)\,e^{i\int_0^P \nu_{1E}d\theta}, \quad (5.2')$$

$$[TH_1+TH_2+TE_1]^* = n_{1H}(O)n_{1H}(P)\varkappa_{1H}(P)\,e^{i\int_0^P \nu_{1H}d\theta} + n_{2H}(O)n_{2H}(P)\varkappa_{2H}(P)\,e^{i\int_0^P \nu_{2H}d\theta} + n_{1E}(O)n_{1E}(P)\varkappa_{1E}(P)\,e^{i\int_0^P \nu_{1E}d\theta}. \quad (5.3')$$

Integration in (5.2) and (5.3) is carried out with respect to the shortest arc of great circle connecting points O and P, since distortion of "beams" TH_1, TH_2, and TE_1 is small.

Replacement of true trajectory of beam by geodesic is permissible if path of beams TH_1, TH_2, and TE_1 form angle of more than 10° with twilight belt. In case of paths for which beams slip along twilight belt, for exact calculations it is necessary to consider distortion of beams due to refraction and also change of coefficients of excitation at point of radiation n(0) and at point of reception n(P) due to lens effect.

All parameters entering in formulas (5.2)-(5.3) can be determined from graphs given in Chapter IV according to data on earth's surface and lower layers of ionosphere along calculated route. Data on lower layers of ionosphere are determined from near field of very long waves by method presented in Chapter III.

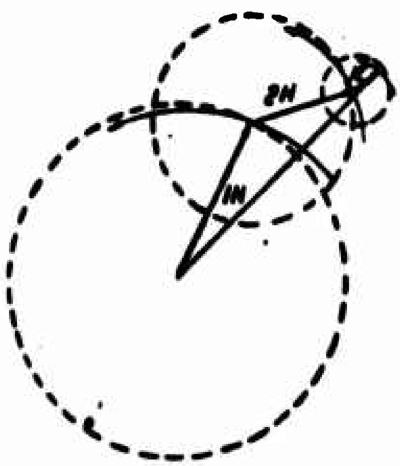


Fig. 23.

After all characteristics of normal waves on route are determined, calculation of fields E_r, E_θ, E_φ, H_r, H_θ, H_φ by formulas (5.2)-(5.3) is carried out by analytic or graphic method. In Fig. 23 is depicted vector diagram, obtained through use of graphic method.

For E_r basic role is played by TH_1 and TH_2 waves. With increase of distance θ, all vectors of waves revolve counterclockwise and decrease in magnitude. Thus each of them forms trajectories in the form of twisted spirals. In view of fact that $\alpha_{1H} > \alpha_{2H}$, angular velocity of rotation of TH_1 waves is more than that of TH_2 waves. Therefore, moving in revolving system of coordinates, together with vector TH_1, we shall see rotation of vector TH_2 clockwise with angular velocity determined by difference $\alpha_{1H} - \alpha_{2H}$. Here total amplitdue and its phase with respect to wave TH_1 will oscillate.

§ 2. Spatial Dependencies of Amplitude and Phase of Distant Field (Uniform Routes)

A. VLW Propagation by Day

We shall start with simplest case of VLW propagation, when route passes above marine surface and is illuminated evenly by sun. In this case, according to Chapter III, following parameters of medium apply:

$$a = 6370 \text{ км}, \ \varepsilon_e = \infty, \ h_1 \simeq 70 \text{ км}, \ \omega_{0.1}^2 = 0{,}3 \cdot 10^{12},$$
$$\nu_{eff}^{(1)} = 2 \cdot 10^7, \ h_2 \simeq 75 \text{ км}, \ \omega_{0.2}^2 = 2 \cdot 10^{12}, \ \nu_{eff}^{(2)} = 10^7,$$
$$\omega_H = 8 \cdot 10^6.$$

At distance of more than 2500 km, for calculation with accuracy of 10%, it is possible to disregard all normal waves except TH_1. Therefore calculation formula (5.1) for E_r takes simple form:

$$E_r = -iA_0\frac{a^2}{r^2}\sqrt{\frac{W}{\sin\theta}}\, n_{1H}^2 D_{\nu_{1H}}(a', b)\, D_{\nu_{1H}}(a', r)\, e^{i\left(\nu_{1H}\theta + \frac{\pi}{4}\right)}. \tag{5.4}$$

Field E_φ, owing to smallness of $\varkappa_{1H}$ is practically absent. If radiating Hertz doublet and receiving point are on surface of earth, that is, b = a and r = a, then, since D(a', a) = 1, formula (5.4) will be simplified still further:

$$E_r = -iA_0\sqrt{\frac{W}{\sin\theta}}\, n_{1H}^2 e^{i\left(\nu_{1H}\theta + \frac{\pi}{4}\right)}, \tag{5.5}$$

where θ is angular distance between points O and P.

In expanded form instantaneous value of E_r will be

$$E_r e^{-i\omega t} = -iA_0\sqrt{\frac{W}{\sin\theta}}\, n_{1H}^2 e^{-\beta_{1H}\theta} e^{-i\left(\omega t - \alpha_{1H}\theta - \frac{\pi}{4}\right)}, \tag{5.6}$$

whence amplitude of distant field $|E_r|$ is equal to

$$|E| = A_0\sqrt{\frac{W}{\sin\theta}}\,|n_{1H}^2|\, e^{-\beta_{1H}\theta} \tag{5.7}$$

In Fig. 24 are given dependencies of $|E_r|$ on angular distance θ for W = 1 kilowatt, for three frequencies f = 12, 15, and 19 kilocycles. In the same place are given for comparison results of measurements of daytime field from round-the-world expedition of 1921-1922 under summer conditions [43].

From Chapter III it follows that daytime summer conditions differ little from daytime conditions in period of equinox; therefore it is permissible to compare results of calculation and experiment. Phase of distant VLW field in daytime, as follows from (5.6), grows evenly with distance for $a\theta > 2000$ km.

On distances less than 1500 km it is necessary to consider higher types of waves. In Fig. 25 is presented analytic curve of $|E_r(\theta)|$, taking into account quasi-TH_1, quasi-TH_2, and higher types of waves. In the same place is given experimental dependence of $|E_r(\theta)|$, taken with help of aircraft flying on route from England (340 km from Rugby) to Cairo [45] in the summer of 1950. Since results of work [45] are presented in relative magnitudes, then on basis of good coincidence of theory and experiment for distant field (Fig. 24) we assumed that both curves coincide for large values of θ. Here also is revealed good coincidence of experiment with theory.

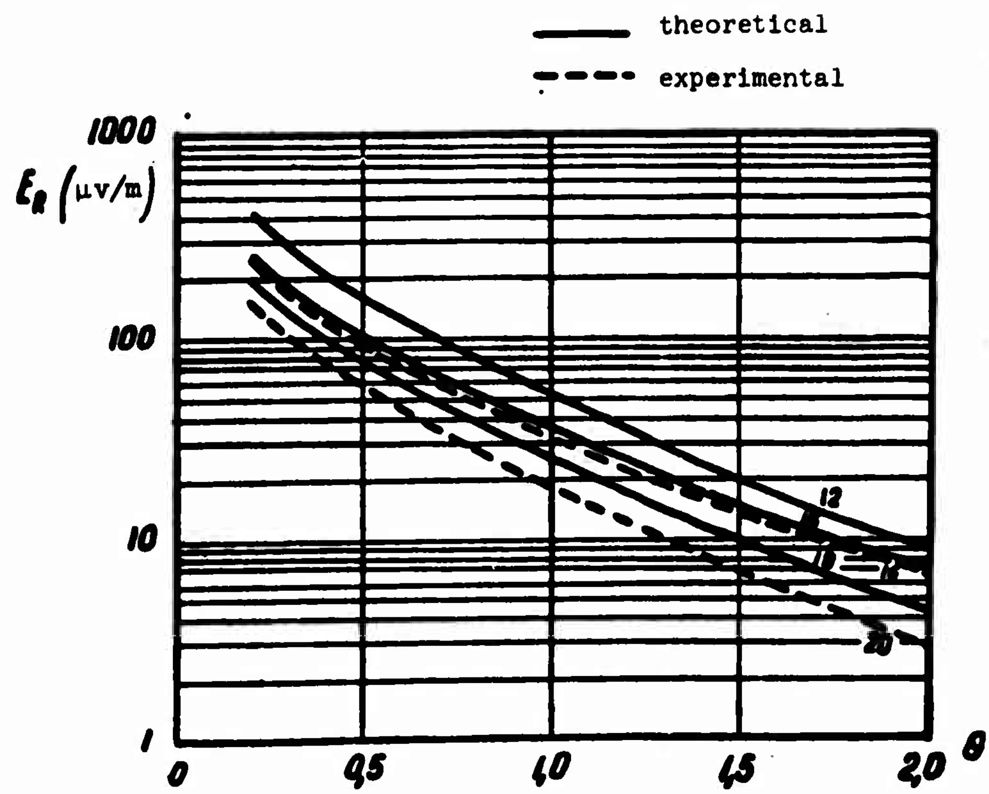


Fig. 24.

It is necessary to note that in existing literature on long waves are widely applied the phenomenological formulas of Osteen-Kogan, Fuller, Ekspenshid, and others. All of them have following structure:

$$|E_r| = \frac{377HI}{\lambda D} e^{-\frac{a}{\lambda^m} D}, \tag{5.8}$$

where $D = a\theta$ — linear distance,

$P = HI$ — moment of Hertz doublet,

λ — wavelength, $m = \frac{1}{3} - 1$.

First factor of (5.8) determines field of dipole in free space, the second considers damping in medium. If this formula is compared to (5.7) and we consider

$$\frac{a}{\lambda^m} = \frac{\beta_{1H}}{a}, \tag{5.9}$$

$$\frac{377HI}{\lambda a\theta} = A_0\sqrt{\frac{W}{\sin\theta}} \cdot |n_{1H}|, \tag{5.10}$$

then sharp distinction in dependencies of first and second factor on frequency ω is immediately apparent. According to Fig. 18, β_{1H} depends on frequency, and weakening of distant field with frequency (Fig. 24) is caused by fall of $|n_{1h}|$ (Fig. 19). In phenomenological formulas (5.8), conversely, pre-exponential factor

increases with frequency, and weakening of distant field with frequency should be provide by second factor. Thus slope of drop of phenomenological curves $|E_r(\theta)|$ is increased with frequency. From Fig. 24 it follows that slope is practically independent of frequency. Therefore phenomenological formulas give grossly incorrect quantitative and qualitative results. Unfortunately they occupy central place in all monographs and textbooks on propagation of radio waves in sections on long waves, misleading broad radiotechnical circles.

For VLW propagation in daytime above land it is necessary to consider conductivity of earth σ_0. Formulas for calculation of field above land surface are not changed, and in earth or under surface of water will be

$$E_r(H,\theta) = E_r(a,\theta)\left(\frac{k_1}{k_0}\right)^2 e^{-k(1-i)H} \text{ etc.,} \tag{5.11}$$

where $E_r(a, \theta)$ is determined by formula (5.6).

If route is nonuniform with respect to ionosphere and ground conductivity, then, according to (5.2)-(5.3), it is necessary to replace exponential factor $\nu_{1H}\theta$ by integral and quantity n_{1H}^2 by product $n_{1H}(0)\cdot n_{1H}(P)$, where in coefficients $n_{1H}(0)$ and $n_{1H}(P)$ it is necessary to consider ground conductivity near radiator and receiver. Formula for E_r takes form

$$E_r = -iA_0\sqrt{\frac{W}{\sin\theta}}\cdot n_{1H}(0)\cdot n_{1H}(P)\, e^{i\left(\int_0^\theta \nu_{1H}\,d\theta + \frac{\pi}{4}\right)}. \tag{5.12}$$

As can be seen "takeoff" and "landing" sites for radio waves play considerably smaller role here the route itself. In this respect very long waves differ essentially from medium waves, investigated by group of Soviet scientists under leadership of academicians Mandelstam and Papaleksi. It is necessary to note also the distinction in shore refractions. On VLW there is real shore refraction, nonvanishing with distance from shore, but it is extraordinarily weak, as can be seen from insignificant increase of $\Delta\alpha_{JH}$ during transition through land-sea boundary (Fig. 15).

Influence of conductivity of land, predicted by theory within limits of errors of measurements, agrees with experimental investigations. In above cited work [43] are described changes of daytime field strength of radio station in San Francisco (f = 23 kilocycles), taken on ship proceeding from England to New Zealand through Panama Canal. Field strength was increased by approximately twice during transition from radioroute from San Francisco to Panama Canal (almost entirely land) to

radioroute from San Francisco to the Pacific Ocean (15° S, 100° W) (entirely sea). From Figs. 15 and 18 we determine by means of extrapolation that β_{1H} for sea ($\sigma_0 = \infty$) is equal approximately to 2.2. For land of average conductivity, taking into account frequency correction, $\Delta\beta_{1H} \simeq 0.8$. Hence $E_{r(\text{суша})}/E_{r(\text{море})} \simeq \frac{1}{2}$. It is necessary to note that in Fig. 8 of work [43] on directions of routes are plotted field strength reduced to 5000 km.

B. VLW Propagation at Night

Now we shall consider VLW propagation for period of vernal equinox under conditions when entire route passes through nonilluminated region above marine surface. Disregarding irregular variations of parameters of lower layers of ionosphere, caused by turbulence and other factors, we shall consider that along entire night route medium, according to Chapter III, is characterized by following parameters:

$$a = 6370 \text{ км}, \ \sigma_0 = \infty, \ h_1 = 86{,}5 \text{ км}, \ \omega_{0,1}^2 = 0{,}3.10^{12},$$
$$\nu_{\text{eff}}^{(1)} = 8.10^6, \ h_2 = 90 \text{ км}, \ \omega_{0,2}^2 = 1{,}5.10^{12}, \ \nu_{\text{eff}}^{(2)} = 5.10^6,$$
$$\omega_H = 8.10^6.$$

For distances of more than 2000 km, for calculation of field strength with accuracy of 10%, it is possible to be limited to three waves: quasi-TH_1, quasi-TH_2, and quasi-TE_1.

In Figs. 26 and 27 are given amplitude dependencies on distances $|E_r(\theta)|$ and dependence of phase on distance, $\arg E_r(\theta)$, for case $b = a$ and $r = a$ with above-mentioned parameters of medium. Calculation was conducted with first formula of (5.2).

Characteristics of normal waves ν_j and n_j are determined on graphs of Chapter IV.

Letters A, B, and C on Figs. 26 and 27 denote curved pertaining respectively to frequencies f = 13 kc, 16 kc, and 20 kc.

First of all, let us consider amplitude dependencies (Fig. 26). Oscilliatory variations of amplitude of total field E_r, as are those of all other components, are caused by interference of TH_1 and TH_2 waves. Quasi-TE_1 wave has phase velocity close to that of quasi-TH_1 wave and considerably smaller amplitude. Therefore it gives long-period shallow pulsations. On small distances TH_2 wave dominates; on large TH_1 dominates. Cause of this is large coefficient of excitation and attenuation factor of TH_2 wave.

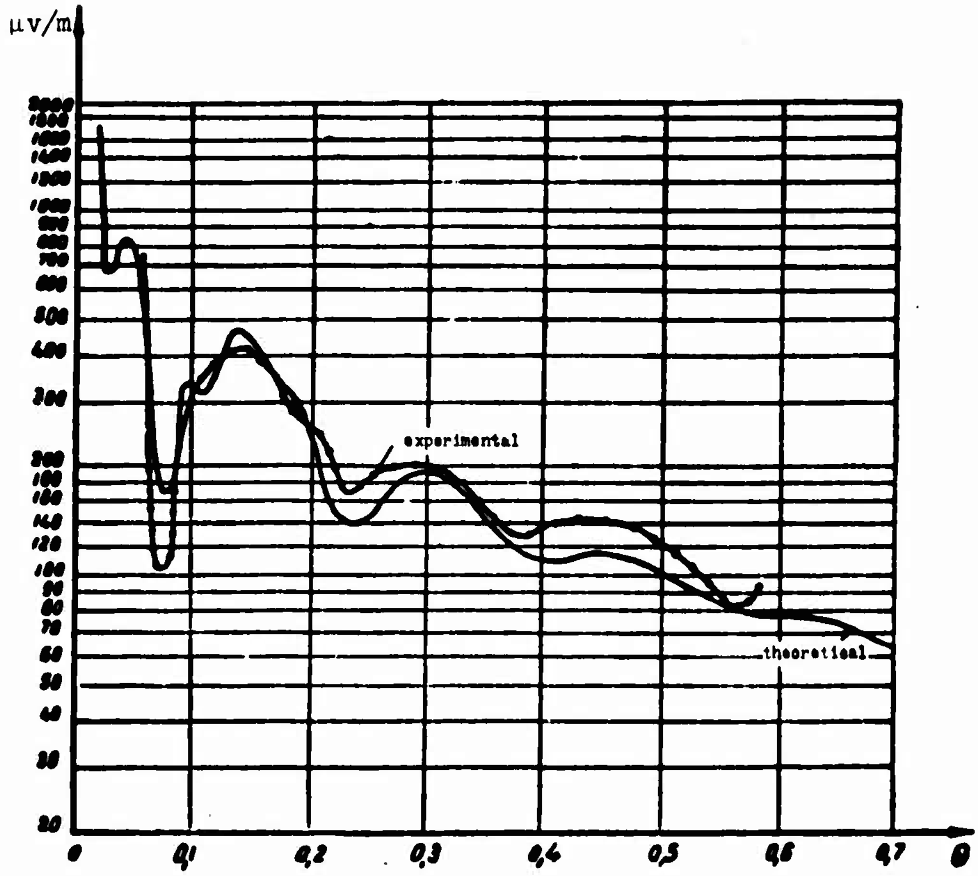


Fig. 25.

In Fig. 28 are given distances θ_{min} and θ_{max} at which amplitude of E_r turns into minimum (solid curves) and maximum (dotted curves). Figures on the right denote reference numbers of extrema. In interval of angular distances marked by heavy lines TH_1 and TH_2 waves are approximately equal; therefore low points of $|E_r|$ attain greatest depth here. These regions θ we call $\theta_{min\ min}$.

As can be seen from Fig. 28, for certain frequencies there exist two deep adjacent minima of approximately identical magnitude. Dotted curve shows dependence of $[E_r]$ on frequency at $\theta_{min\,min}$. From Fig. 28 it follows also that with increase of frequency ω, region $\theta_{min\ min}$ rapidly departs from radiator and depth of low points increases.

In upper part of Fig. 28, above band $\theta \approx \theta_{min\,min}$, quasi-$TH_1$ wave dominates; in lower part quasi-TH_2 wave dominates. At large distances, when $(\theta - \theta_{min\,min})$ is great, amplitude of pulsations of curves $|E_r|$, depicted in Fig. 26, attenuates, and night field of $|E_r|$ may be calculated by monomial formula (5.6).

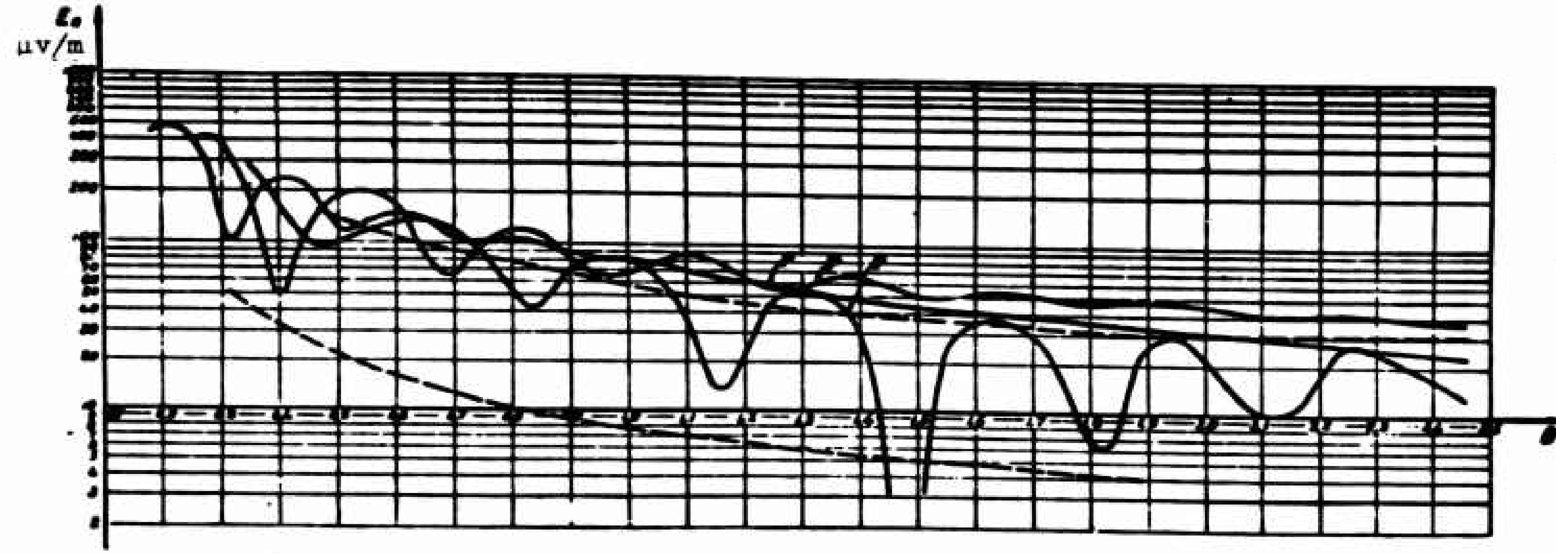


Fig. 26.

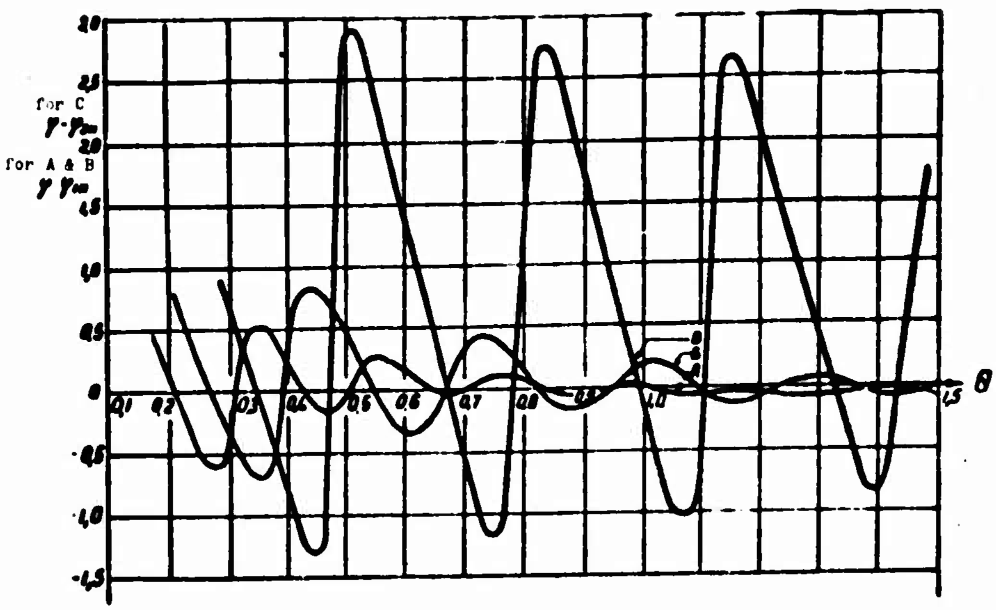


Fig. 27.

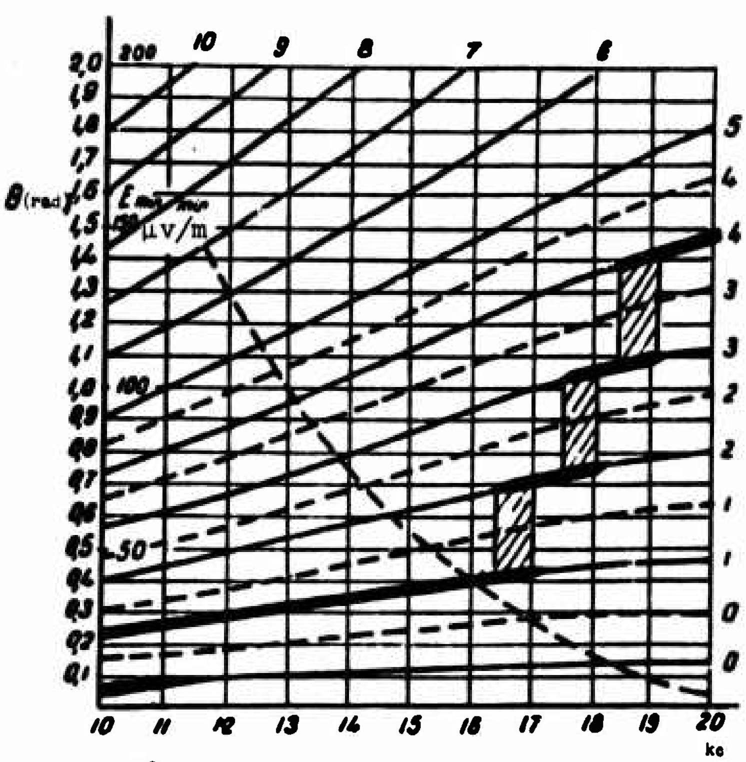


Fig. 2F.

Mean amplitude of $|E_r|$ for averaged pulsations at small distances $\theta < \theta_{\min\min}$ is determined by formula (5.6), where as basis is taken TH_2 wave. Mean amplitude of $|E_r|$ for $\theta > \theta_{\min\min}$ is determined by formula (5.6), where as base is taken TH_1 wave. Since attenuation factors of TH_2 and TH_1 waves are different, mean amplitude $|E_r|$ drops for small values of θ faster than for large ones. Therefore monomial phenomenological formulas are not useful for description of space regularities of night field in whole interval of θ.

Complicated interference character of night field very much hampered experimental investigations of space regularities of VLW. Therefore in work [43] are given only quasi-minimum and quasi-maximum values of $|E_r|$ for frequencies of 16-30 kilocycles, plotted in Fig. 26 by dotted line.

Despite the fact that mean amplitude of night field, determined by TH_2 wave at distances $\theta < \theta_{\min\min}$ and by TH_1 wave for $\theta > \theta_{\min\min}$, is larger than day values (Fig. 24) in regions of θ close to $\theta_{\min\min}$, real night field can be considerably less than day. Example of this is sharp fall of distant field of radio station GBR (Rugby), f = 16 kilocycles, during reception in Moscow. From Figs. 24 and 26 it follows that $\frac{E_{r(\text{день})}}{E_{r(\text{ночь})}} \cong \frac{150}{50} = 3$, which coincides well with experimental swing of diurnal variations of station GBR (§ 3, Chapter V), which is located at angular distance of $\theta = 0.4 \cong \theta_{\min\min}$ from Moscow.

Another example of fall of night field, which was earlier considered an anomaly, is Kauka (Hawaiian Islands) - Tokyo route, with extent of $\theta \cong 1$ radian (frequency, 17.7 kilocycles). From Figs. 26 and 24 it follows that when $\theta = 1$ radian, $|E_r|$ falls in region $\theta_{\min\min}$, where $\frac{E_{r(\text{день})}}{E_{r(\text{ночь})}} \cong \frac{40}{20} = 2$, which coincides with duirnal swing of variations of $|E_r|$ given in book by Acad. A. N. Shchukin [1].

Thus anomalous daily variation of distant field of VLW is always obtained for regions $\theta \cong \theta_{\min\min}$, where there occurs interchange of TH_2 and TH_1 waves. It is interesting to note that in range of 10-20 kilocycle ratio $E_r(\text{день})/E_r(\text{ночь})$ in region $\theta \cong \theta_{\min\min}$ does not depend on frequency and equals approximately 3.

In regions $\theta < \theta_{\min\min}$ and $\theta > \theta_{\min\min}$ day field is less than night. This case, naturally, is observed more frequently in practice of long-distance communication than with the preceding. Therefore it is considered normal. Examples of normal cases are:

1. Route from Rocky point (United States) to New Southgate (England), distance $\theta \cong 0.85$ radian (frequency, 17.130 kilocycles). On it is observed regular increase of night field

$$\left.\frac{E_r(\text{день})}{E_r(\text{ночь})}\right|_{\text{эксп.}} \approx \frac{90}{180} = 0.5.$$

From Fig. 28 it follows that at f = 17 kilocycle in region $\theta = 0.85$ there is interference maximum.

From Figs. 26 and 24 it follows that

$$\left.\frac{E_r(\text{день})}{E_r(\text{ночь})}\right|_{\text{теор.}} \cong 0{,}4.$$

2. Route from Annapolis (United States) to Moscow, distance $\theta = 1.22$ radian (frequency f = 19 kilocycles; NSS); on it in periods of equinox is observed regular increase of field at night (§ 3 of this Chapter)

$$\left.\frac{E_r(\text{день})}{E_r(\text{ночь})}\right|_{\text{эксп.}} = 0{,}32 \div 0{,}35.$$

From Figs. 26 and 28 it follows that $\theta = 1.22$ for f = 19 kilocycles is between third maximum and minimum of interference curve $|E_r|$ and ratio

$$\left.\frac{E_r(\text{день})}{E_r(\text{ночь})}\right|_{\text{теор.}} \cong 0{,}3.$$

3. Route from Naven (Germany) to Tokyo, distance $\theta = 1.4$ radian (frequency f = 16.5 kilocycles); on this route there regularly is observed 5.5-6-fold gain of night field as compared to day [1].

From Fig. 28 one may see that for f = 16.5 kc, $\theta = 1.4$ radian, at night there is a fourth interference maximum. According to Figs. 24 and 26

$$\left.\frac{E_r(\text{день})}{E_r(\text{ночь})}\right|_{\text{теор.}} \cong \frac{13}{65} = \frac{1}{5}.$$

4. Route from Hawaiian Islands to Moscow, distance $\theta = 1.8$ radian (frequency 17.5 kilocycles); on this route was observed 4-fold increase of night field (authors' measurement). From Fig. 28 it follows that for $\theta = 1.8$ radian and frequency of 17.5 kilocycles at night there is fifth interference maximum; field in it is of the order of 30 microvolts/m (Fig. 26). By day (Fig. 24) field constitutes 7 microvolts/m. Thus

$$\left.\frac{E_r(\text{день})}{E_r(\text{ночь})}\right|_{\text{теор.}} = \frac{1}{4{,}3}.$$

We could have multiplied number of examples of anomalous and normal daily variation of $|E_r(t)|$, which, within limits of errors of experiment, are contained in above-stated theoretical calculations.

It is necessary to note that diurnal swing of $|E_r|$, determined by ratio $\frac{E_{r(\text{день})}}{E_{r(\text{ночь})}}$, does not depend on conductivity of earth along route, since corrections $\Delta\beta_{1H}$ and $\Delta\beta_{2H}$ are identical both for day and night. Therefore during comparison of experiment with theory, we used data for $\sigma_0 = \infty$.

Now let us turn to discussion of graph of night phase depicted in Fig. 27. In it are given phase differences between total vector of field E_r and vector of basic wave, having the biggest modulus (Fig. 23). Graphs A, B, and C pertain respectively to frequencies of 13, 16, and 20 kilocycles. Since in interval of angular distances $0.2 < \theta < 1.5$ for f = 13 and f = 16 basic wave, according to Figs. 26 and 28, is quasi-TH_1 wave, while for f = 20 kilocycle it is TH_2 wave, then for graphs A and B on ordinate are plotted phase differences between vector of total field and that of TH_1 wave, while for graph C ordinate bears vector of total field and of TH_2 wave.

From Fig. 27 one may see that deviations of phase $\Delta\varphi$ of total field from basic wave oscillates around zero value and attenuates with growth of θ. Maxima of deviation of phase $\Delta\varphi$ from zero value are shifted with respect to maxima of amplitude of field $|E_r|$ by 1/4 period of space variations, that is, are observed at various angular distances. With increase of frequency f swing of oscillations $\Delta\varphi$ grows.

At our disposal there are no experimental data on measurement of phase of distant field. Therefore confidence in results of Fig. 27 can settle only on coincidence of experimental data on amplitude of field with calculated values. The latter were obtained from the same formulas used for data on phase.

§ 3. Temporal Dependencies (Diurnal Variation) of Amplitude and Phase of Distant VLW Field on Nonuniform Routes of Variable Illuminance

For practical purposes it is very important to know diurnal variations of amplitude and phase of distant VLW field. In this section we shall consider two cases:

1) anomalous daily variation on Rugby (England) - Moscow route (θ = 0.4, f = = 16 kilocycles),

2) normal daily variation on Annapolis (United States) - Moscow route (θ = 1.22, f = 19 kilocycles) in period of vernal equinox.

Regular heterogeneities along route are:

1) change of conductivity of earth σ_0,

2) changes of properties of ionosphere, caused by shift of twilight belt.

Regular change of phase and amplitude of distant VLW field at fixed point of reception, in this case in Moscow, appears only because of shift of twilight belt along route.

Calculation of field of E_r at the surface of earth was made by formula (5.2), where, instead of angular distance, is introduced average time on route t by the formula

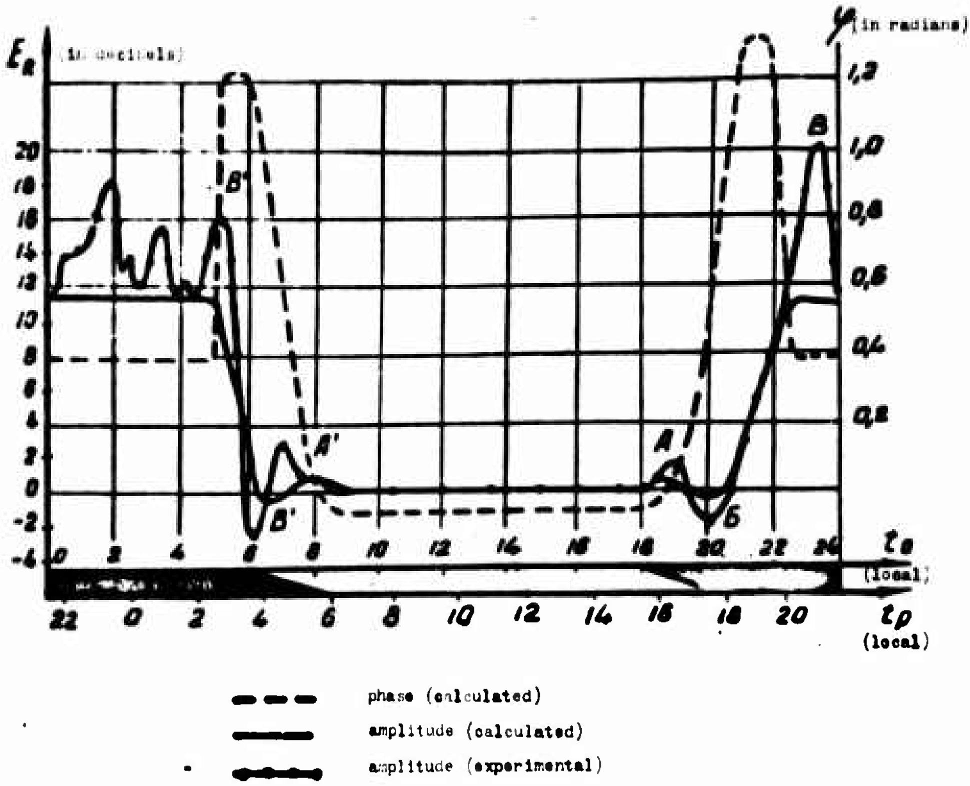
$$d\theta = \frac{\theta_0}{t_0} dt. \tag{5.13}$$

Here θ_0 is angular distance between O and P, t_0 is difference in local times between O and P. Integration is carried out from t to t + t_0, where t is time at point of reception P.

Dependencies of characteristics of normal waves ν_j, n_j, and $\varkappa_j$ on time were determined with help of graph of heights $h_1(t)$, Fig. 9, on which abscissa t was replaced by θ per formula (5.13) and from Figs. 17-22 for night and day values of ν_j, n_j, and $\varkappa_j$. Values of ν_j, $\varkappa_j$, and n_j in twilight period were obtained from Figs. 17-22 by means of linear interpolation. Since curve $h_1(t)$ consists of straight lines, then integrals entering in (5.2) are calculated by formulas for areas of triangles.

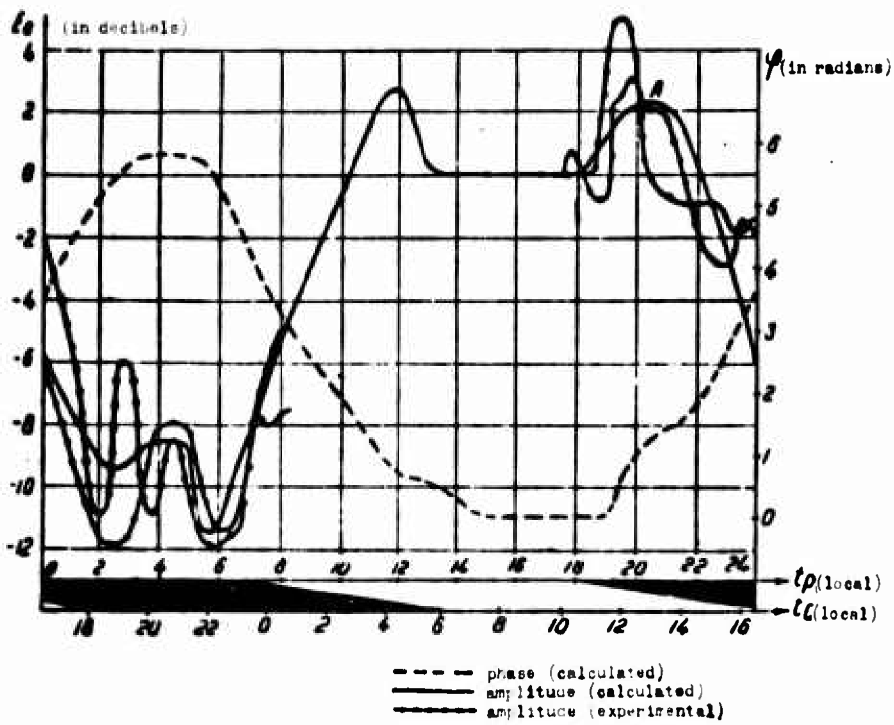
In Fig. 29 are given computed values of amplitude and phase of field of station GBR (Rugby) in Moscow for period of vernal equinox. In the same place is given experimental curve of daily variation of amplitude, taken on 19 March 1955 in region of Moscow. All data on amplitude are given in decibels with respect to daytime values of field, which is distinguished by high stability of amplitude.

As can be seen from Fig. 29, there is good coincidence between calculations and experiment. Theory, in virtue of simplified initial data (Fig. 9), considers only regular oscillations of field. Therefore from comparison of experimental and calculated curves it is possible to dismember regular and irregular parts of field. This will be done in following section. Here we are interested only in regular changes of daily variation.



(in decibels)
(in radians)
(local)
phase (calculated)
amplitude (calculated)
amplitude (experimental)

Fig. 29.



(in decibels)
(in radians)
(local)
(local)
phase (calculated)
amplitude (calculated)
amplitude (experimental)

Fig. 30.

On the basis of presented theory it is simple to explain temporal variation of amplitude of field E_r, depicted in Fig. 29. In course of day amplitude E_r does not change, since field at point of reception is caused by one quasi-TH_1 wave. Small drop of amplitude after sunset (point A, Fig. 29) is due to amplification of adhesion effect for TH_1 wave, due to rise of ionosphere at point of reception. This drop has been observed by a number of authors, but until now has not been explained [1, 2, 3, 4, 5].

With advance of twilight westward, an even greater part of route is submerged in darkness. Therefore field of E_r, determined by TH_1 wave, is strengthened, since attenuation of TH_1 wave at night is less than by day (point B). Only after twilight reaches England and conditions close to those of night have been established data point of radiation does TH_2 wave start to reach point of reception, and, owing to mutual extinguishing of TH_1 and TH_2 waves, total field drops. The biggest drop of E_r (point C) is observed under conditions for which TH_1 and TH_2 are in opposite phase. In period of ascent of sun, along route the picture is approximately reversed.

Coincidence of daily variation for calculated and experimental amplitudes forces us to assume that calculated curve of daily variation of phase is close to reality. The biggest deviation of phase from daytime value occurs in periods of sunsets and sunrises, when vectors of TH_1 and TH_2 (Fig. 23) are perpendicular.

In Fig. 30 are presented calculated curves of daily variation of amplitude and phase of field of station NSS (Annapolis) as received in Moscow for period of vernal equinox. In the same place are given experimental curves for amplitude taken in Moscow on 19 and 20 March 1955.

In daytime at point of reception quasi-TH_1 wave dominates. Evening drop at point A caused by adhesion effect, is more sharply expressed here than in preceding case. Amplification of field at sunrise at point of reception and after sunset at point of radiation is caused by interference of TH_1 and TH_2 waves. In these period there occurs exchange of leading waves. At night quasi-TH_2 wave dominates, since $\theta < \theta_{\min\min}$ (Figs. 26 and 28).

Divergence of calculated and empirical curves, as follows from comparison of two empirical curves obtained on different days is caused by irregular processes in ionosphere. Diurnal swing of phase of NSS is more than 4 times more than in preceding case (GBR), which is caused by corresponding increase of distance and frequency.

§ 4. Reception of VLW Under Water

For radiator located on land (b = a) and receiving point immersed at depth H = a - r under land or under water, formulas (5.1) for fields taking following simple form:

$$E_r(H,\theta) \cong \frac{E_r(a,\theta)}{60\lambda\sigma_0} e^{-2\pi\sqrt{\frac{30\sigma_0}{\lambda}}(1-i)H-\frac{\pi i}{2}}, \tag{5.14}$$

$$H_\varphi(H,\theta) \cong H_\varphi(a,\theta) e^{-2\pi\sqrt{\frac{30\sigma_0}{\lambda}}(1-i)H}, \tag{5.15}$$

$$E_\theta(H,\theta) = -\frac{E_r(a,\theta)}{\sqrt{60\lambda\sigma_0}} e^{-2\pi\sqrt{\frac{30\sigma_0}{\lambda}}(1-i)H+i\frac{\pi}{4}}, \tag{5.16}$$

$$H_r(H,\theta) = H_r(a,\theta)\cdot e^{-2\pi\sqrt{\frac{30\sigma_0}{\lambda}}(1-i)H}, \tag{5.17}$$

$$E_\varphi(H,\theta) = E_\varphi(a,\theta)\cdot e^{-2\pi\sqrt{\frac{30\sigma_0}{\lambda}}(1-i)H}, \tag{5.18}$$

$$H_\theta(H,\theta) = H_\theta(a,\theta) e^{-2\pi\sqrt{\frac{30\sigma_0}{\lambda}}(1-i)H}, \tag{5.19}$$

where $E_r(a, \theta)$, $H_\varphi(a, \theta)$, $H_r(a, \theta)$, $E_\varphi(a, \theta)$, and $H_\theta(a, \theta)$ are total components of electromagnetic field above land surface, calculated by formulas (5.2) and (5.3).

λ – wavelength in free space (in meters),

σ_0 – conductivity of water or soil $\frac{1}{(\text{ohms./meter})}$.

With error of the order of factor $\left(1 - \frac{\nu_j}{k_1 a}\right)$, it is possible to consider that in air

$$-E_r = H_\varphi \cdot 120\pi;\ E_\varphi = H_r \cdot 120\pi.$$

Therefore on graphs (Figs. 26 and 27) for fields of $E_r(a, \theta)$, given in Chapter V, it is easily possible to determine $E_r(H, \theta)$, $H_\varphi(H, \theta)$, and $E_\theta(H, \theta)$ in earth by changing scales on ordinates accordingly:

$$\frac{1}{60\lambda\sigma_0}e^{-2\pi\sqrt{\frac{30\sigma_0}{\lambda}}H} \quad \text{for } E_r,$$

$$\frac{1}{120\pi}e^{-2\pi\sqrt{\frac{30\sigma_0}{\lambda}}H} \quad \text{for } H_\varphi, \tag{5.20}$$

$$\frac{1}{\sqrt{60\lambda\sigma_0}}e^{-2\pi\sqrt{\frac{30\sigma_0}{\lambda}}H} \quad \text{for } E_\theta.$$

Remaining components in this work are not given, and according to the formula (5.1), depend on polarization factor $\varkappa_j$.

Since for sea water mean value $\sigma_0 \cong 4 \frac{1}{\text{ohm/meter}}$, then for middle of range of VLW ($\lambda = 20{,}000$ m) parameter $60\lambda\sigma_0 = 4.8 \cdot 10^6$. Therefore, according to (5.14) and (5.16),

$$\frac{E_r(H,\theta)}{E_\theta(H,\theta)} = \frac{1}{\sqrt{60\lambda\sigma_0}} = 0{,}46.10^{-3}, \text{ that is}$$

component E_θ by more than 2000 times exceeds component E_r.

Since $\varkappa_{1H}$ does not drop below $\frac{1}{100}$ by day and does not exceed 1/10 at night, component E_φ constitues $\sim\frac{1}{10} E_\theta$ at night and $\sim\frac{1}{100} E_\theta$ by day, that is, is always greater than E_r. From these data the conclusion can be made that the most effective underwater and underground antennas will be systems reacting to component E_θ, equivalent to component H_φ.

Emf induced by distant VLW field in vertical loop directed towards radiator is determined by formula

$$E_{ind} = -\frac{d\Phi}{dt} = -i\omega\mu_0 H_\varphi(a,\theta) \iint_S e^{-\delta H}\, dH dx, \tag{5.21}$$

where

$$\delta = 2\pi\sqrt{\frac{30\sigma_0}{\lambda}}(1-i); \; \mu_0 = 4\pi \cdot 10^{-7} \text{ h/m}. \tag{5.22}$$

Integral (5.21) is selected with respect to area of loop. E_{ind} is due basically to component E_θ. If loop has rectangular form, with side l parallel to surface of earth and with side d normal to it, then

$$E_{ind} \cong -\frac{2i\omega l}{\delta}\mu_0 H_\varphi(a,\theta)\, e^{-\delta H_0} \cdot \sinh\left(\frac{\delta d}{2}\right), \tag{5.23}$$

where H_0 is vertical coordinate of center of loop.

With loop of sufficiently small dimensions $\left(\frac{\delta d}{2} \ll 1\right)$

$$E_{ind} = -i\omega\mu_0 H_\varphi(a,\theta) S e^{-\delta H_0} = E_{ind}(a,\theta) e^{-\delta H_0}, \tag{5.24}$$

where $S = ld$ is area of loop,

$E_{ind}(a, \theta)$ is emf in loop located above ground.

Magnitude of δ for $\lambda = 20{,}000$ m and $\sigma_0 = 4$ is equal ~ 0.5, that is, damping with submersion of loop is ~1/2 neper per meter of depth or 4 db per meter. In loop located in vertical plane perpendicular to [beam] direction, in radiator will be induced emf caused by component E_φ (or H_θ). From above-cited data it follows that E_{ind} of such loop will be 1/100 (by day) and 1/10 (at night) of E_{ind} of loop located parallel to beam.

Significant result of theory expounded here is fact that field at depth H under surface of water (land) depends only on field on surface of water (land) at point with same geographic coordinates. From Chapter V it follows that field on surface of water can be calculated with great accuracy. Therefore, using conversion formulas (5.20), we determine with the same degree of accuracy fields under water and land.

From calculations given in Chapter V it follows that to provide reliable communication on VLW, it is necessary to consider regularly sharp drops in signal strength observed in certain intervals of distances, depending on frequency of signals and also on seasonal and diurnal variations of ionosphere.

Designations

A — scalar electrical potential,

A_k — scalar electrical potential of spherical k layer,

A_j — scalar electrical potential of normal wave of number j,

$A_0 = 0.1829 \cdot 10^{-20}$; $A_0' = \frac{A_0}{120\pi}$,

A_1 — coefficient of focusing of beam,

a — radius of earth,

B — scalar magnetic potential,

B_k — scalar magnetic potential of spherical k layer,

B_j — scalar magnetic potential of normal wave of number j,

$\begin{vmatrix} B \\ A \end{vmatrix}$ — potential vector function,

$\begin{vmatrix} B_k \\ A_k \end{vmatrix}$ — potential vector function for spherical k layer,

b — radial coordinate of Hertz doublet,

c — velocity of light = 3.10^{10} cm/sec,

$_{,j}(x, y) \ldots D_{\nu_j}(x', y')$ — analytic functions of two arguments, composed of cylindrical Bessel and Neumann functions of order ν_j and their derivatives,

D_0, D_1, D_2 — lengths of paths of beam,

Д, D — distance; name of lower layer of ionosphere,

$\overline{D}$ — diameter of heterogeneity in ionosphere,

$D_{BB} \ldots D_{\Gamma B}$ — indices of refraction and conversion of beams in ionosphere,

$\left\| D^{(1,2)} \right\|$ — matrices of refractive index in 1-th and 2-nd layers of ionosphere,

d — distance between routes of waves,

$\overline{E}$ — electric field strength,

E_r; E_φ; E_θ — components of electric field strength in spherical coordinates,

E_r^k; E_φ^k; E_θ^k — components of electric field strength in layer of number k,

E_{ind} — [emf] (ЭДС) induced in loop,

e — charge of electron,

$F_{r,\theta,\varphi}$ — operator determining field according to data on medium and exciting [factors] of number k,

f — frequency of oscillations,

H_r; H_φ; H_θ — component of magnetic field strength in spherical coordinates,

H_r^k; H_φ^k; H_θ^k — components of magnetic field strength in layer of number k,

$\overline{H}_0$, H_0 — intensity of constant magnetic field of Earth,

$H = a - r$ — depth of submersion of point of reception P under surface of sea water or land,

H_0 — depth of submersion of center of receiving loop under surface of water or land,

h — instantaneous coordinate of height of ionosphere, measured from surface of earth,

h_1; h_2 — height 1-st and 2-nd layer of ionosphere, measured from surface of earth,

$\overline{h}$ — averaged height of turbulent ionosphere,

$\overline{I}$ — density of outside current,

I_r — radial component of density of outside current,

I_j — component of radial component of density of outside current I_r in normal wave of number j,

Jm — imaginary part of quantity,

$\begin{vmatrix} I \\ 0 \end{vmatrix}$ — vector function of current density,

j_ν — modified Bessel function of order ν,

j — reference number of normal wave,

k — reference number of spherical layer of medium,

k_0 — wave number in earth,

$k_1 = \frac{\omega}{c}$ — wave number in atmosphere,

L_r — operator of normal waves on coordinate r,

L_r^* — conjugate operator of normal waves,

L_θ — operator of wave equation on coordinate θ,

$L_{\theta\varphi}$ — operator of wave equation on coordinates θ and φ,

$L^{(1)}_{\nu_j}$; $L^{(2)}_{\nu_j}$ — Legendre-Hankel functions of order ν_j of first and second kind,

l_r — differential-matrix expression generating operator L_r,

l_θ — differential expression generating operator L_θ,

M — constant finite quantity,

m — mass of electron,

N_e — concentration of electrons,

N_j — normalizing factor for normal wave of number j,

n_j — coefficient of excitation of normal wave of number j,

n_{jE}; n_{jH} — coefficients of excitation of quasi-TE_j and quasi-TH_j waves,

n_ν — modified Neumann function of order ν,

O — point of radiation,

P — point of reception,

P — electrical moment of Hertz doublet,

P — pressure,

P_{ν_j} — first fundamental function of Legendre's equation of order ν_j,

Q_{ν_j} — second fundamental function of Legendre's equation of order ν_j,

Re — real part of quantity,

R_{BB} ... R_{TB} — indices of reflection and conversion during reflection of beams from layers of ionosphere,

r — radial coordinate,

r_{BB}; $r_{\Gamma\Gamma}$ — indices of reflection of beam from Earth,

S — area of loop,

S_j — function of eikonal of normal wave,

T — temperature,

TE — waves with transverse electrical polarization,

TH — wave with transverse magnetic polarization,

t — time,

U — component of eigenvector of conjugate operator of normal waves,

$U_j(\theta, \varphi)$ — amplitude of modulated normal wave of number j,

$\begin{vmatrix} U \\ V \end{vmatrix}$ -- vector function of conjugate operator of normal waves,

V — component of eigenvector of conjugate operator of normal waves,

$\bar{v}$ — velocity of electrons,

v_c — velocity of propagation of signal,

W — emissive power,

Y; Z — components of vector function $\begin{vmatrix} Y \\ Z \end{vmatrix}$,

Y_{jk}; Z_{jk} — components of eigenvector $\begin{vmatrix} Y \\ Z \end{vmatrix}$ of operator of normal waves in k layer,

α — azimuthal angle,

α — angle of incidence of beam on Earth,

α_j — real part of wave number of normal wave of number j,

α_{jH}, α — real parts of wave numbers of quasi-TH_j and quasi-TE_j waves,

β — angle of incidence of beam on ionosphere,

β_j — attenuation factor of normal wave of number j,

β_{jH}; β_{jE} — attenuation factors of quasi-TH_j and quasi-TE_j waves,

γ — angle of slip of beam upon striking ionosphere,

Δ and δ — increments and errors,

Δ_j — Wronskian for Legendre's equation,

δ — attenuation factor of radio waves in land and in water,

ε — complex dielectric constant,

ε_0 — complex dielectric constant of Earth,

$\varepsilon_{rr}^{(k)}$, $\varepsilon_{\theta\theta}^{(k)}$.... — components of tensor of complex dielectric constant in layer of number k,

θ — angular spherical coordinate,

$\varkappa_j$ — polarization factor of normal wave of number j,

$\varkappa_{jH}$, $\varkappa_{jE}$ — polarization factors of quasi-TH_j and quasi-TE_j normal waves,

λ — wavelength,

μ_j — eigenvalue of conjugate operator,

μ — small parameter,

$\nu^*_j = \nu_j + \frac{1}{2}$ — complex wave number of normal wave of number j,

ν_{jH}; ν_{jE} — complex wave numbers of quasi-TH_j and quasi-TE_j normal waves,

ν_{eff} — number of collisions for electrons,

$\nu_{eff}^{(k)}$ — number of collisions in k layer,

$\xi = \cos\theta$,

$\pi \cong 3.1416$,

Σ — summation operator,

σ — specific electrical conductivity,

σ_0 — conductivity of earth,

τ — time or propagation time of signal,

τ — parameter of anisotropy of ionosphere,

Φ_j — phase factor of forced normal waves,

φ — angular spherical coordinate,

φ — phase angle,

χ_j — eigenvalue of number j operator of normal waves,

Ψ_j — phase factor of free normal waves,

ω — angular frequency of oscillations,

ω_0 — critical frequency,

ω_{0k} — critical frequency of k layer,

ω_H — gyromagnetic frequency,

$\frac{\omega_r}{\omega}$ — parameter of reflectance of ionosphere.

д. у. — division of goniometer (0.001 of distance).

Literature

1. Shchukin. Propagation of radio waves, 1940.

2. Dolukhanov. Propagation of radio waves, 1951.

3. Kessenikh. Propagation of radio waves, 1952.

4. Al'pert, Ginzburg, and Feynberg. Propagation of radio waves, 1953.

5. Bremmer. Terrestrial radiowaves, 1949.

6. Anderson. Proceedings of the Institute of Radio Engineers, 1928, Vol. 16, page 297.

7. Round, Eckersley, Tremellen, and Lunnon. Journal of the Institution of Electrical Engineers. 1925, 63, page 933.

8. Osteen. Proceedings of the Institute of Radio Engineers. 1922-1931, series of articles.

9. Ekspenshid, Anderson, and Bailey. Proceedings of the Institute of Radio Engineers. 1926 January. Bell. Syst. Techn. Journ. 1925 July.

10. Kitchen, Bressi, and Tremellen. The Proceedings of the Institution of the Electrical Engineers. 1953, Part III, page 100.

11. Interavia. 1954, Vol. 9, No. 6, pages 377-378.

12. Regiment and Watson. The Proceedings of the Institution of the Electrical Engineers. 1947, Vol. 94, Part IIIa, page 1016.

13. Rayleigh. Proceedings of the Royal Society. 1904, Vol. 72, page 40.

14. Poincaré. Proceedings of the Royal Society. 1904, Vol. 72, page 42.

15. MacDonald. Proceedings of the Royal Society. 1903, Vol. 71, page 251; 1914, A90, page 50.

16. Nicholson. Philosophical Magazine. 1910, Vol. 19, page 516, page 757, Philosophical Magazine. 1910, Vol. 20, page 157, Philosophical Magazine, 1911, Vol. 21, page 62, page 281.

17. Sommerfeld. Annalen der Physik. 1909, Vol. 28, page 665.

18. Love. Roy. Soc. Phil. Trans. A. 1915, Vol. 215, page 105.

19. Marx. Annalen der Physik. 1912, Vol. 37, page 29.

20. Rybchinskiy. Annalen der Physik, 1916, Vol. 41, page 191.

21. Smith. Journal of the Institution of Electrical Engineers, 1933, Vol. 73, page 574. Journal of the Institution of Electrical Engineers, 1934, Vol. 9, page 22.

22. Watson. Proceedings of the Royal Society, 1918, Vol. 95a, page 83.

23. Watson. Proceedings of the Royal Society, 1919, Vol. 95, page 546.

24. Riedbeck. Trans. Chalmers Univ, 1944, No. 34.

25. Kenric. Physical Review, 1928, Vol. 31, page 1940.

26. Weirich. Annalen der Physik, 1928, Vol. 85, page 5.

27. Brekhovskiy and Ryazin. Izvestia Acad. Sciences of USSR (physics series) 1946, X, No. 3, page 285.

28. Budden. Philosophical Magazine, 1951, Vol. 42, page 1.

29. Budden. Philosophical Magazine, 1952, Vol. 43, page 1179.

30. Booker. Proceedings of the Royal Society, 1935, Vol. 150, page 267.

31. Breswell, Budden, Radcliffe, et al. The Proceedings of the Institution of the Electrical Engineers, 1951, Vol. 98, Part III, page 221.

32. Streicher. The Proceedings of the Institution of the Electrical Engineers, 1955 March, Part C, No. 1.

33. Bain, Breswell, et al. The Proceedings of the Institution of the Electrical Engineers, 1952 April, Part IV.

34. Breswell. The Proceedings of the Institution of the Electrical Engineers, 1952 April, Part IV.

35. Best, Radcliffe, and Wilkins. Proceedings of the Royal Society, 1936, Vol. 156, page 614.

36. Krasnushkin. Method of normal waves in application to waveguides and their algebraic symbols, Doctoral dissertation, Moscow State University, November 1945.

37. Krasnushkin. Method of normal waves in application to problem of long-range radio communication, Moscow State University, 1947.

38. Bates and Messi. Journal of Atmospherical and Terristrial Physics, 1951, Vol. 2, page 1. Journal of Atmospherical and Terristrial Physics, 1952, Vol. 2, page 253.

39. Mitra. Journal of Geophysical Research, 1951, Vol. 56, page 373.

40. Nicolet. Journal of Atmospherical and Terristrial Physics, 1953, Vol. 3, page 200.

41. Naymark. Linear differential operators, State Publishing House of Theoretical and Technical Literature, 1952.

42. Rytov. Transactions of Physics Institute of the Academy of Sciences, 1940, Vol. II, Issue I, page 41.

43. Tremellen. Marconi Review, 1950, Vol. 13, page 153.

44. Breswell and Streicher. Monthly Notices of the Royal Astronomical Society, 1949, Vol. 109, page 28.

45. Budden. Phylosophical Magazine, 1953, Vol. 44, page 504.

46. Baumler. Proceedings of the Institute of Radio Engineers, 1924, Vol. 12, page 5.

47. Harwood. The Proceedings of the Institution of the Electrical Engineers, 1954, Vol. 100, Part III, No. 70.

48. Breswell. Journal of Atmospherical and Terristrial Physics, 1952, Vol. 2, page 226.

49. Breswell and Bain. Journal of Atmospherical and Terristrial Physics, 1952, Vol. 2, page 216.

50. Bureaw. L'onde Electric, 1947 February, No. 239.

51. Budden, Radcliffe, and Wilkins. Proceedings of the Royal Society, 1939, Vol. 171, page 188.

52. Hollingworth. Journal of the Institution of Electrical Engineers, 1926, Vol. 64, page 579.

53. Ukhov. Navigation. Publishing House of water transport, 1954.